Physics PhD qualifying examinations

1967-1980

University of California Irvine

Posted by Jeff Schmidt, author of *Disciplined Minds*

NEW STYLE Comprehensive/Qualifying Examination

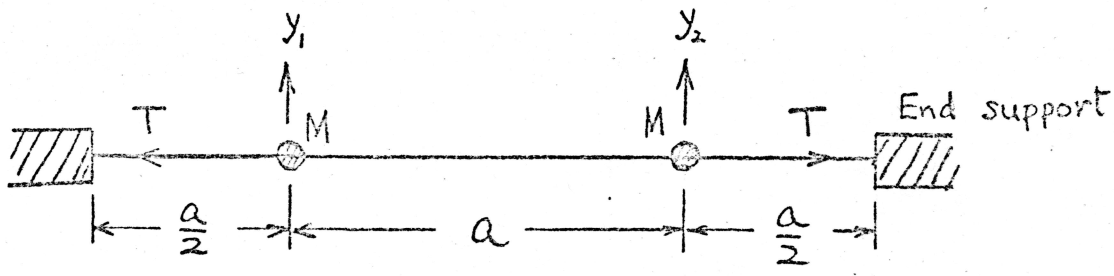
Paper I. May 18, 1967 9 a.m. - 12 noon

Choose two problems out of each of the three sections:

Section A

Section A.

1.



Two equal masses (M) are attached to a weightless string under tension (T) exerted by rigid end supports. The masses are spaced as shown. Consider only transverse vibrations in the plane of the paper (as measured by y_1 and y_2).

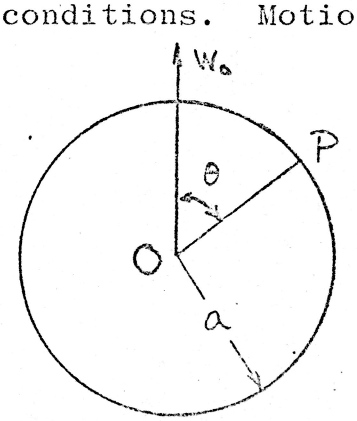
a) Set up the equations of motion for the two masses in terms of y_1 and y_2.

b) Determine the normal frequencies of the system.

c) Find a set of normal coordinates applicable to the system. Give the "separated" equations of motion for these coordinates (Y_1 and Y_2).

d) If mass (1) is clamped in the position shown ($y_1 = 0$), then what frequency of transverse vibration will mass (2) have?

2. A sphere of mass (M) and radius (a) rotates freely about its midpoint (O) at angular velocity (w_o), under force-free conditions. Motion of a point (P) on the surface is suddenly stopped, as by impact. (As shown, point (P) is at an angle (θ) to the original axis of rotation).



2. a) What is the angular momentum of the sphere just before impact? Just after impact?

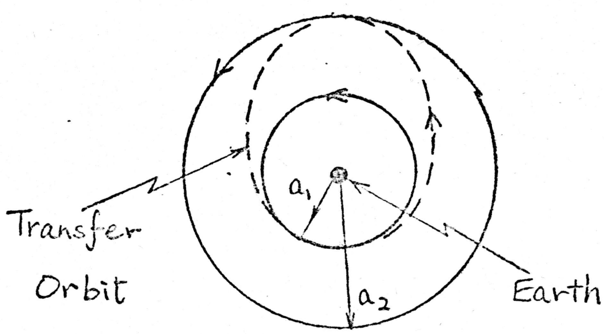
b) Find the magnitude and direction of the angular velocity ($\vec{w}$) immediately following impact. Hint: Useful components are w_r (along O-P) and w_t (tangent to the surface at P).

c) Point (P) is then released immediately after stopping and the sphere left force-free. What is then the translational velocity of the sphere?

d) Give the energy of sphere: (i) before the impact at (P), (ii) while (P) is instantaneously motionless, and (iii) after release (part (c)).

of inertia

Recall that the moment of inertia of a solid sphere about the midpoint is $I_o = \frac{2}{5} Ma^2$.

3. 

A satellite originally in circular earth orbit at radius (a_1) is elevated to a high circular earth orbit of radius (a_2).

The "transfer orbit" for this purpose is an ellipse (see dashed orbit) tangent at each extreme to one of these orbits.

a) What well-known relationship exists between total energy (E) (kinetic plus potential) and the major axis of any orbit?

b) Using this relationship show that

$$\frac{1}{E_1} + \frac{1}{E_2} = \frac{2}{E_{xtr}}$$

where E_1 is the total energy for circular orbit (1),
E_2 " " " " " " " (2),
and E_{xtr} " " " " " the "transfer orbit."

3. c) Which orbit change requires greater expenditure of energy, "boosting" into the transfer orbit at radius (a_1) or out of it at radius (a_2).

d) If ΔE_1 is the energy expenditure at radius (a_1) and ΔE_2 that at radius (a_2), express the ratio $\frac{\Delta E_1}{\Delta E_2}$ as a function of (a_1) and (a_2).

Section B. You may use either the rationalized MKS system or the mixed Gaussian units but specify which.

4. A parallel plate condenser consists of two circular plates of radius r at a distance h apart, with h small compared with r. The condenser is filled with a dielectric of dielectric constant κ, which is imperfect, i.e. slight conducting with conductivity σ. The condenser is initially charged to a potential U between its plates and isolated.

a) Find the capacitance of the condenser and the resistance of the dielectric disc.

b) Find the charge on the positive plate at time t.

c) Find the Maxwell displacement current through the dielectric.

d) Find the magnetic field inside the dielectric.

5. Given the charge density and current density distributions, deduce the integral solutions of the vector and scalar potentials from the Maxwell equations.

A small plane loop of area a is fixed in position and carries a current $I_o \sin \omega t$ where t is the time. Find the electric and magnetic fields in the radiation zone (i.e. far away from the loop, compared with the linear dimension of a as well as the wavelength of light at circular frequency ω.)

(To be continued on next page.)

5. (cont'd.)

[Identities that might be useful:

$$\text{curl curl } \underset{\sim}{Y} = \text{grad div } \underset{\sim}{Y} - \nabla^2 \underset{\sim}{Y}$$

$$\text{curl}(\underset{\sim}{A} \wedge \underset{\sim}{B}) = (\underset{\sim}{B} \cdot \nabla)\underset{\sim}{A} - (\underset{\sim}{A} \cdot \nabla)\underset{\sim}{B} + \underset{\sim}{A} \text{ div } \underset{\sim}{B} - \underset{\sim}{B} \text{ div } \underset{\sim}{A}]$$

6. A long circular cylindrical magnet of radius a has a uniform magnetization M perpendicular to the axis of the cylinder. Find the magnetic intensity and magnetic induction in the cylinder and outside. Find the magneto-static energy of the system per unit length of the cylinder.

Sec Section C.

7. Answer the following briefly:

a) Someone asserts that Einstein's postulate (that the speed of light is not affected by the uniform motion of the source or the observer) must be discarded because it violates "common sense." How would you refute him?

b) A circularly polarized beam of light in vacuum impinges on the plane surface of an isotropic dielectric with refractive index n at an angle of incidence arc tan n. What is the polarization of the reflected ray?

c) Why does the sky appear blue and the sun red?

d) When a crystal of Iceland spar is placed over some printed letters, the image appears double. Explain.

e) Devise a way to identify the polarizing direction of a sheet of Polaroid.

8. a) A ray of light impinging on one side of a thin slab of dielectric will produce a succession of parallel rays emerging from the other side. Deduce and discuss in detail the interference fringes these rays can be made to produce.

b) Explain the principle and the use of <u>either</u> the Fabry-Perot interferometer <u>or</u> the Michelson's interferometer.

NEW STYLE Comprehensive/Qualifying Examination Paper I.

9. a) Prove the Green's theorem:

$$\int_V (\Psi\nabla^2\phi - \phi\nabla^2\Psi)\,dV = \int_S (\Psi\nabla\phi - \phi\nabla\Psi)\cdot d\underset{\sim}{S}$$

where the volume V is bounded by the surface S and $d\underset{\sim}{S}$ is, by convention, pointing outwards.

b) Consider the propagation of a monochromatic electromagnetic wave with angular frequency kc in vacuum. c is the speed of light in vacuum. Show that, if U denotes a Cartesian component of the electric field and S is a closed surface enclosing no sources of radiation then at a point P inside the surface S,

$$U(P) = \frac{1}{4\pi}\int_S \left[\frac{e^{ikr}}{r}\nabla U - U\nabla\left(\frac{e^{ikr}}{r}\right)\right]\cdot d\underset{\sim}{S}$$

where $\underset{\sim}{r}$ is the vector from P to a point on the surface S. [This is known as the Kirchhoff theorem].

c) Show how Huygen's principle can be used to give the diffraction pattern on a screen placed behind an opague screen with a small aperture illuminated by a monochromatic point source.

Sketch the justification of Huygen's principle in this situation using Kirchhoff's theorem, stating carefully the approximations used.

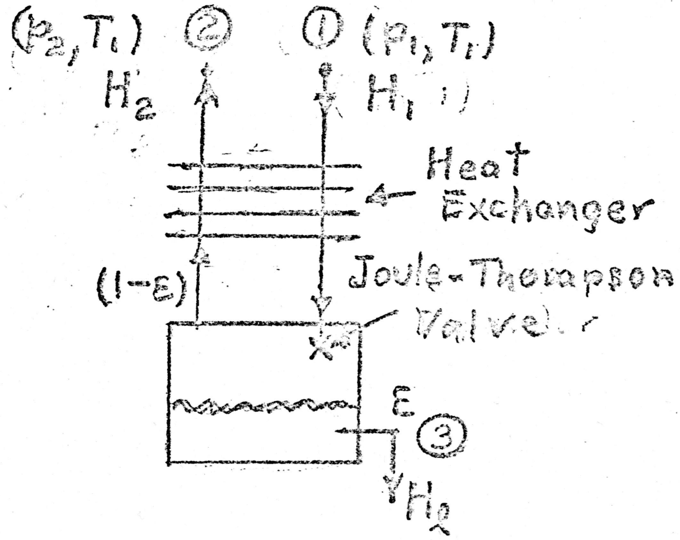
NEW STYLE COMPREHENSIVE/QUALIFYING EXAMINATION

Paper II. May 18, 1967 2 p.m. - 5 p.m.

Choose two problems out of each of the three sections.

Section A.

1. The air liquefier represented in the accompanying sketch operates as follows. Compressed air at pressure (p_1) and room temperature (T_1) enters at point (1). This incoming air is then cooled at constant pressure (p_1) by passage through a heat exchanger, as shown. The resultant cold air next expands irreversibly to atmospheric pressure through a Joule-Thompson throttling value, (see figure). The portion remaining vapor then becomes heated by return at constant atmospheric pressure through the heat exchanger to (essentially) room temperature at point (2).



a) Explain why the enthalpy (H) is the thermodynamic function which is conserved in this case? Recall that enthalpy is defined as

$$H = E + pV$$

where E is the internal energy of the gas and p and V the pressure and volume.

x (ε) b) For steady state operation, some fraction (ε) of the incoming vapor will condense and may be withdrawn at point (3) as liquid. Set up the enthalpy balance equation of the overall system for this condition.

(ε) c) Solve result (b) to obtain the fraction (ε) of the incoming flow rate removable as liquid.

Obtain the above solutions in terms of the following enthalpies per gram of air:

H_1 is the enthalpy (per gram) of incoming air at point(1) of(p_1,T_1)
H_2 " " " (" ") " outgoing " " " (2) " (p_2,T_1),
and H_3 " " " (" ") " liquid air at (3), as shown.

(NOTE: The heat exchanger merely transfers heat by simple conduction.)

NEW STYLE Comprehensive/ Qualifying Examination Paper II.

3. (cont'd.)

x

x

Du a-raw ov $\vec{F}_i$

a) By treating forces ($\vec{F}_i$) as occurring only through collision with the walls, evaluate the summation as an integral of pressure over the container walls.

b) Treating the pressure as constant over the walls, convert the surface integral to a volume integral over all atoms of the gas.

c) From result b) determine the relationship between kinetic energy density and pressure for the perfect gas.

perfect

d) Suppose an external field force were present (such as a gravitational field). How would that effect evaluation of the summation for a perfect gas?

4. Entrops (S) of a system may be defined as

$$S = K \ln \Omega$$

x ways

where K is Boltzman's constant and Ω is the number of ways of achieving the equilibrium condition. An equivalent definition becomes

$e^{-E/KT}$ slant

$$S = K \frac{2}{2T} T \ln \left(\int e^{-E/KT} d\Gamma\right)$$

where E is the system energy and $d\Gamma$ the volume element in hyper-space* (corresponding to energy (E)0.

a) Using these equivalent definitions show that the number of ways (Ω) of achieving equilibrium is

$e^{\frac{\bar{E}-E}{kT}}$ vc.

$$\Omega = \int e^{\frac{\bar{E}-E}{KT}} d\Gamma$$

Where $\bar{E}$ is the mean energy.

higher up than e

b) What characteristics of the integrand permits easy evaluation? Show thereby that

$$\Omega = \Delta\Gamma$$

for classical statistical mechanics and distinquishable particles?

6-N

*(i.e. 6-N dimensional phase space)

4. c) What does the second expression for entropy (S) become for quantum statistical mechanics and indistinquishable particles (assume N indistinquishable particles).

Section B.

5. Explain the method of steepest descent (the saddle point method) for evaluating the asymptotic value of the definite integral

$$I(z) = \int_c e^{zf(t)} dt$$

for large z.

Hence, prove that, as $z \to \infty$, The Gamma function

$$\Gamma(z+1) = \int_o^\infty t^z e^{-t} dt ,$$

tends asympotically to

$$\sqrt{2\pi} z^{z+\frac{1}{2}} e^{-z} ,$$

and deduce the Stirling approximation for $\ln(n!)$ for large n.

6. Solve the boundary value problem for $u(x,t)$,

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = \sin x , \quad o<x<\pi , \quad t>o .$$

$$u(o,t) = o = u(\pi,t) ,$$

$$u(x,o) = x(\pi-x) .$$

You may leave the answer as a series.

7. Two planes intersecting at right angles are raised to constant potentials V/2 and -V/2 respectively. Calculate the electrostatic field.

NEW STYLE Comprehensive/Qualifying Examination Paper II.

SEC Section C.

8. Explain carefully the important characteristics of each of the following types of solids which distinguish it from the others.

a) A molecular crystal (b) an ionic crystal (c) an intrinsic semiconductor (d) a normal metal (e) a superconductor.

To which of the classes named above, each of the following belongs:

(1) Lithium fluoride (2) Solid methane (3) solid hydrogen (4) Cesium chloride (5) Cu_3Au (6) Rubidium (7) Germanium (8) Indium antimonide (9) Lead (10) Indium

Name two solids which do not belong to any of the five classes named above.

9. Deduce the temperature dependence of lattice specific heat at the low temperature limit and the high temperature limit.

Define Debye temperature. Explain why experimental measurements show that the Debye temperature decreases initially as the temperature is increased from near zero.

10. Define an edge dislocation and a screw dislocation. Explain the role of dislocations in

a) the critical shear stress of pure crystals.
b) the sources and sinks of vacancies in crystals.
c) work-hardening in metals.
d) crystal growth.
e) "whiskers".

NEW STYLE Comprehensive/Qualifying Examination

Paper III. May 19, 1967 9 a.m.-12 noon

Choose two problems out of each of the three sections:

Section A.

1. A neutron is situated in a weak uniform magnetic field which splits the energy levels of the two possible spin states.

   a) If the neutron is initially in the excited (upper) spin state, what is the probability of finding it in this state after a long time?

   b) Suppose there are two neutrons (with non-overlapping spatial wave functions) such that neutron 1 is in the excited spin state and neutron 2 in the unexcited state. What is the probability of finding one neutron in the excited spin state after a long time? (Assume the distance between neutrons is infinitesimal compared to the wavelength of emitted radiation.)

2. A harmonic oscillator of angular frequency ω is in the ground state at t=0. You have at your disposal a weak, spatially uniform, time-dependent force

$$F(t) = 0, \quad t<0$$

$$F(t) = F_0[1-e^{-t/\tau}], \quad t>0$$

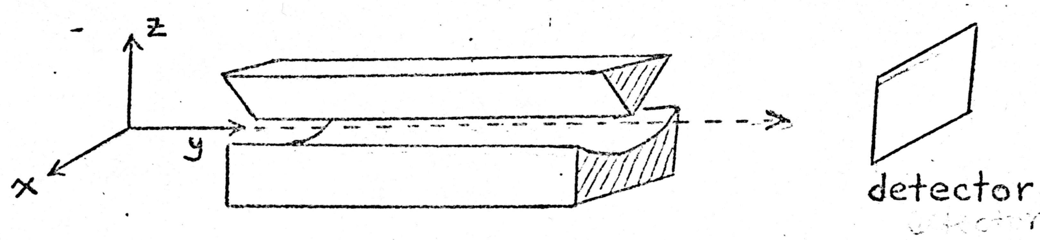
with a time constant τ which may be taken to be either:

(i) $\tau \gg \frac{1}{\omega}$ or

(ii) $\tau \ll \frac{1}{\omega}$

Which time constant would you choose in order to maximize the average probability of finding the system in the first excited state after a long time ($t \gg \tau$)? Find the ratio of the average probabilities for the two conditions above.

3.



A physicist attempts to measure the magnetic moment of a free proton using the Stern-Gerlach apparatus above. In order to define the momentum of the proton beam there is an unavoidable uncertainty in the position along the x-axis. Show, using the Heisenberg relation, that this uncertainty is sufficient to make the experiment impractical.

Section B.

4. Alfred Kastler received the 1966 Nobel Prize in Physics for his work on "optical pumping." Discuss (by means of words, equations, and carefully labeled energy level diagrams) how one optically pumps on the ground state of an alkali atom. (Assume nuclear spin 3/2.)

5.

a) Explain what is meant by "L-S coupling" and "j-j coupling" in complex atoms, and describe the situations in which each is applicable.

b) Explain the origin of the famous Sodium D lines, and describe the Zeeman splitting of the lines in a weak magnetic field.

6.

a) The hyperfine splitting of the ground state of Hydrogen is approximately 1420 megacycles. Estimate the magnetic field within the atom at the point where the proton is situated.

6.(cont'd.)

b) Estimate the relative magnitudes of the level separation, the fine structure splitting, and the hyperfine structure of the Hydrogen atom.

Section C.

7. a) Derive a formula for the energy loss of a non-relativistic charged particle in matter. Show explicitly the dependence on mass, velocity and charge of the incident particle and the relevant properties of the medium.

   b) Given a proton and a deuteron with the same initial velocity in a medium. Which will travel further before stopping? Explain.

8. a) Would the lifetime of a neutron increase or decrease if the neutron-proton mass difference were twice as large as it is? Explain.

   b) Sketch (and justify) the approximate form of the momentum spectrum of the electrons emitted in the beta decay.

9. a) Assuming the ground state spin and magnetic moment of an odd-A nucleus is due entirely to the odd proton or neutron, show that for a given spin, I, there are two possible values for the magnetic moment. Find these 2 values for both odd-Z and odd-N nuclei.

   b) For odd-A nuclei the actual experimental magnetic moments tend to lie between the two values given by assumption (a). What possible causes can you give for this discrepancy?

OLD STYLE Qualifying Examination Paper I.

May 18, 1967 9 a.m.-12 noon

Choose <u>six</u> problems.

1. Given the charge density and current density distributions, deduce the integral solutions of the vector and scalar potentials from the Maxwell equations.

   A small plane loop of area a is fixed in position and carries a current $I_o \sin \omega t$ where t is the time. Find the electric and magnetic fields in the radiation zone (i.e. far away from the loop, compared with the linear dimension of a as well as with the wavelength of light at circular frequency ω.)

   [Identities that might be useful:

   $$\text{curl curl } \underset{\sim}{Y} = \text{grad div } \underset{\sim}{Y} - \nabla^2 \underset{\sim}{Y}$$

   $$\text{curl } (\underset{\sim}{A} \wedge \underset{\sim}{B}) = (\underset{\sim}{B} \cdot \nabla)\underset{\sim}{A} - (\underset{\sim}{A} \cdot \nabla)\underset{\sim}{B} + \underset{\sim}{A} \text{ div } \underset{\sim}{B} - \underset{\sim}{B} \text{ div } \underset{\sim}{A}]$$

2. A long circular cylindrical magnet of radius a has a uniform magnetization M perpendicular to the axis of the cylinder. Find the magnetic intensity and magnetic induction in the cylinder and outside. Find the magnetostatic energy of the system per unit length of the cylinder.

3. a) Prove the Green's theorem:

   $$\int_V (\Psi \nabla^2 \phi - \phi \nabla^2 \Psi) \, dV = \int_S (\Psi \nabla \phi - \phi \nabla \Psi) \cdot d\underset{\sim}{S}$$

   where the volume V is bounded by the surface S and $d\underset{\sim}{S}$ is, by convention, pointing outwards.

3. (cont'd.)

b) Consider the propagation of a monochromatic electromagnetic wave with angular frequency kc in vacuum. c is the speed of light in vacuum. Show that, if U denotes a Cartesian component of the electric field and S is a closed surface enclosing no sources of radiation then at a point P inside the surface S,

$$U(P) = \frac{1}{4\pi} \int_S \left[\frac{e^{ikr}}{r} \nabla U - U \nabla \left(\frac{e^{ikr}}{r} \right) \right] \cdot d\underset{\sim}{S}$$

where $\underset{\sim}{r}$ is the vector from P to a point on the surface S. [This is known as the Kirchhoff theorem.]

c) Show how Huygen's principle can be used to give the diffraction pattern on a screen placed behind an opague screen with a small aperture illuminated by a monochromatic point source.

Sketch the justification of Huygen's principle in this situation using Kirchhof's theorem, stating carefully the approximations used.

4. A uniform electrostatic field E is maintained parallel to a uniform magnetic field B in the same region of space. A fluorescent screen is placed parallel to the parallel fields. Now a beam of β-particles of rest mass m and charge e are injected from a point at a distance L from the screen at right angles towards the screen with various initial speed V (ranging up to almost the speed of light) such that $|\omega \cdot L| << |V|$ where ω is the cyclotron frequency in the magnetic field B. Find the locus of all the particles landing on the screen.

5. Explain the method of steepest descent (the saddle point method) for evaluating the asymptotic value of the definite integral

$$I(z) = \int_C e^{zf(t)}\, dt$$

for large z.

Hence, prove that, as $z \to \infty$, the Gamma function

$$\Gamma(z+1) = \int_0^\infty t^z e^{-t}\, dt,$$

tends asympotically to

$$\sqrt{2\pi}\, z^{z+\frac{1}{2}} e^{-z},$$

and deduce the Stirling approximation for $\ln(n!)$ for large n.

6. Solve the boundary value problem for $u(x,t)$,

$e^{-2t} \sin x$

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = e^{-2t} \sin x, \quad 0<x<\pi, \quad t>0$$

$$u(0,t) = 0 = u(\pi,t),$$

$$u(x,0) = x(\pi-x).$$

You may leave the answer as a series.

7. Two planes intersecting at right angles are raised to potentials V/2 and -V/2 respectively. Calculate the electrostatic field.

OLD STYLE Qualifying Examination Paper II.

May 19, 1967 9 a.m.-12 noon

Choose <u>six</u> problems

1. A neutron is situated in a weak uniform magnetic field which splits the energy levels of the two possible spin states.

   a) If the neutron is initially in the excited (upper) spin state, what is the probability of finding it in this state after a long time?

   b) Suppose there are two neutrons (with non-overlapping spatial wave functions) such that neutron 1 is in the excited spin state and neutron 2 in the unexcited state. What is the probability of finding one neutron in the excited spin state after a long time? (Assume the distance between neutrons is infinitesimal compared to the wavelength of emitted radiation.)

2. A harmonic oscillator of angular frequency ω is in the ground state at $t=0$. You have at your disposal a weak, spatially uniform, time-dependent force

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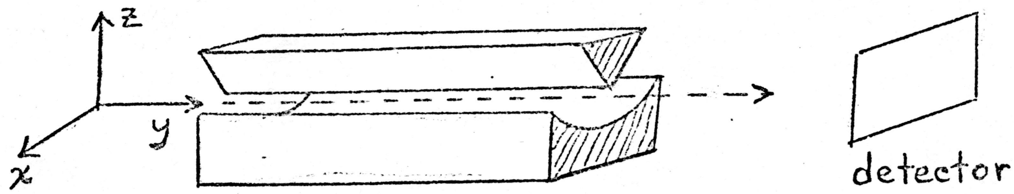
with a time constant τ which may be taken to be either:

(i) $\tau \gg \frac{1}{\omega}$ or

(ii) $\tau \ll \frac{1}{\omega}$

Which time constant would you choose in order to maximize the average probability of finding the system in the first excited state after a long time ($t \gg \tau$)? Find the ratio of the average probabilities for the two conditions above.

3.



A physicist attempts to measure the magnetic moment of a free proton using the Stern-Gerlach apparatus above. In order to define the momentum of the proton beam there is an unavoidable uncertainty in the position along the x-axis. Show, using the Heisenberg relation, that this uncertainty is sufficient to make the experiment impractical.

4. Using the variational principle, estimate the ionization energy of the Helium atom. You may use Hydrogen-like trial wave functions, and the following aids:

$$\phi_{100} = \sqrt{\frac{Z^3}{\pi a_o^3}}\, e^{-Zr/a_o}$$

$$\int \frac{e^{-(r_1+r_2)}}{|\vec{r}_1-\vec{r}_2|}\, d\vec{r}_1 d\vec{r}_2 = 20\pi^2$$

5. a) Define the helicity operator for a relativistic particle of spin $\frac{1}{2}$.

b) Construct the four-component solutions of the free Dirac Equation which are eigenstates of helicity.

c) Show that in the extreme relativistic limit these solutions are eigenstates of $\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4$

d) If the matrix element for a beta decay is

$$M \sim \Psi_e^+(1 + \gamma_5)\Psi_{\nu_e}$$

What does this imply about the polarization of the electron?

OLD STYLE Qualifying Examination Paper II. May 19, 1967

6. Describe the assumptions which lead to the Thomas-Fermi model of complex atoms. Derive the basic relevant equations and use them to find the Z-dependence of (a) The radial size of an atom, and (b) the average kinetic energy of an electron.

7. Suppose that the effective potential which describes the interaction of a very slow neutron with a Fe atom in a block of magnetized iron is given by

$$V(\vec{r}) = A\,\delta(\vec{r}) - \vec{\mu}_n \cdot \vec{H}\, f(\vec{r})$$

Here the first term represents the interaction with the nucleus and the second term gives the effective interaction of the neutron's magnetic moment with the average magnetic field, $\vec{H}$, of the atom. (Suppose $f(\vec{r})$ is a function which is unity inside a unit cell containing the atom and zero outside).

a) Find an expression for the scattering cross-section, in Born Approximation, for the two different orientations of the neutron's spin.

b) Given a block of iron of length L, how can it be used to obtain a polarized beam of neutrons from an incident unpolarized beam? Find an expression for the net polarization,

$$P \equiv \frac{I_+ - I_-}{I_+ + I_-}$$

where I_+, I_- are the beam intensities for spin up and down, respectively.

1. Recent observations suggest that the universe is filled with microwave black-body radiation at a temperature $T_0 = 3°K$, which probably originated in the "big-bang". As seen in the local inertial frame, S, which is at rest with respect to nearby galactic clusters, the specific intensity of the radiation is isotropic and is that which one expects inside a black-body cavity of $T_0 = 3°K$:

$$I_\nu = \frac{2h\nu^3}{e^{h\nu/kT_0} - 1}$$

what is the specific intensity, as a function of direction seen by an observer who moves with velocity $\vec{v}$ with respect to the frame S?

2. Two reference frames have the same acceleration with respect to an inertial frame S therefore their relative velocity is constant. Can we use the Lorentz transformation between them? Discuss.

3. Show that the physically meaningful spherically time independent part of a spherically symmetric field satisfying the Klein-Gordon equation, is the so-called Yukawa potential and find the plane wave expansion for such a potential.

4. Determine the motion of a system with two degrees of freedom expressed by the two generalized Langrange coordinates q_1 and q_2 and with a kinetic energy T and a potential energy V given by

$$T = \frac{1}{2}\left(q_1^2 + q_2^2\right)\left(\dot{q}_1^2 + \dot{q}_2^2\right)$$
$$V = \left(q_1^2 + q_2^2\right)^{-1}$$

5. Consider an atom with two optical electrons.

a) The spectra of such a system is usually described in terms of LS or jj coupling. Comment briefly on the two types of coupling, and on what determines their relevancy in a given atom.

b) Assuming the coupling is pure jj, show that the Zeeman splitting of an energy level in a magnetic field B is given by

$$E = E_0 + \mu_0 g_J B M_J$$

where

E_0 is the energy of the unsplit level,

μ_0 is the Bohr magneton,

$\hbar M_J$ is the component of J, the total angular momentum, on B,

and g_J is the Lande' g factor for j-j coupling and is given by

$$g_J = \frac{1}{2}(g_1 - g_2)\left[\frac{j_1(j_1+1) - j_2(j_2+1)}{J(J+1)}\right] + \frac{1}{2}(g_1 + g_2).$$

where j_1 and j_2 are the total angular momenta of the optical electrons, and g_1 and g_2 are the corresponding g factors.

c) Calculate g_1 for the case $j_1 = \ell_1 + \frac{1}{2}$

6. Consider the scattering of a particle from the potential

$$V(\underset{\sim}{r}) = a^3 V_0 \delta(|\underset{\sim}{r}| - a).$$

Show that by carrying out a partial wave decomposition, one may explicitly solve the integral equation for the Schrödinger wave function. Exhibit the form of the solution in the asymptotic region of large $|\underset{\sim}{r}|$.

Discuss the following points:

a) When the energy of the incident particle is low, show that only s-wave scattering is important. What does one mean by "low energy" in this statement? If you have not obtained the explicit solution, feel free to discuss the question for a general potential of finite range.

b) When the energy of the incident particle is high (compared to what?), show the scattering cross section is given accurately by the first Born approximation. Again, this may be discussed for a general, short range potential if the complete solution has not been obtained.

c) Given the scattering amplitude associated with some potential for the ℓ^{th} partial wave as a function of energy (for all energies) how does one deduce the energies of possible bound states of this value of ℓ?

Using the specific form derived in the first part of this problem, discuss the occurance of bound states for the delta function shell when $V_0 < 0$. Also state in a general way the criterion for the occurances of scattering resonances in the ℓ^{th} partial wave.

cont'd next page.

6. Cont'd.

Useful Identities

$$\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}) = \sum_{l=0}^{\infty} (2l+1) i^{l} j_l(-kr) P_l(-\hat{k}\cdot\hat{r})$$

$$\int_{-1}^{+1} d\mu \, P_l(-\mu) P_{l'}(-\mu) = \frac{2}{2l+1} \delta_{ll'}$$

$$P_l(\hat{n}\cdot\hat{n}') = \frac{4\pi}{2l+1} \sum_{m=-l}^{+l} Y_{lm}^{*}(\hat{n}) Y_{lm}(\hat{n}')$$

$$Y_{l0} = \left(\frac{2l+1}{4\pi}\right)^{1/2} \cdot P_l \, .$$

7. Derive expressions for the wavefunction, ψ, and energy, $\mathcal{E}$, of a Bloch electron in a static magnetic field, H. (Work in the effective mass approximation and choose H along the z direction. Also for convenience use the Landau gauge $\vec{A} = [0, Hx, 0]$. Obtain the relation between the center of the orbit X_0 and the momentum of the electron. Sketch the energy versus momentum curves ($\mathcal{E}$ vs $\hbar\vec{k}$). Now consider the situation of a Bloch electron in crossed static magnetic and electric fields (H_z and E_x respectively). Again derive expressions for ψ and $\mathcal{E}$. Discuss the physical nature of the terms in the expression for $\mathcal{E}$ which are linear and quadratic in the electric field.

8. Derive, by means of classical equations of motion, etc., the expression for the polarization, $\vec{P}$, induced by an electromagnetic $\vec{E}$ field and the corresponding dielectric constant, $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ of a medium consisting of a "simple cubic" array of electric dipole oscillators. Assume that the electric dipole

oscillators are made up of two particles, having masses m_+ and m_- and electric charges +e and -e, held together by a "spring type" force $F = -m\omega_0^2 x$. Include a damping force $F = m\gamma\dot{x}$ and take into account the Lorentz dipolar field. (Hint: Use a slab configuration with the direction of propagation of the EM radiation normal to the slab faces and assume that $\exp i(\vec{q}\cdot\vec{r} - \omega t) \approx \exp(-i\omega t)$, where $\vec{q}$ is the wave vector of the radiation.

Sketch curves of $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ versus ω. Also sketch the curves of ω vs q which characterize the propagation of the EM radiation in the medium. Finally discuss the transmission of EM radiation through a very thin slab of the medium frequency ranges (a) $\omega < \omega_T$, (b) $\omega_L < \omega < \omega_T$ and (c) $\omega_T < \omega$ where ω_L is the longitudinal resonance frequency i.e. the frequency at which $\epsilon_1(\omega) = 0$, and ω_T is the transverse resonance frequency i.e., the frequency at which $\epsilon_2(\omega)$ is a maximum.

CLASSICAL PHYSICS

Ph.D. Qualifying Exam
(Closed Book)

May 16, 1968
9 AM - 12 NOON

This exam is divided into 3 sections. Do 2 problems from each section.

A. Do 2 problems.

A1. A point charge is brought from infinity to a position near an infinite grounded plane conductor. If the induced surface charge on the conductor is now "frozen" in place and the point charge is removed, what is the energy in the remaining electric field?

A2. Consider a plane, polarized, monochromatic wave of frequency ω, normally incident on a metallic surface of conductivity σ. Compute the pressure exerted by the wave on the metal by

(a) either determining the Lorentz force on the currents induced in the metal surface

(b) or applying momentum conservation to the problem.

A3. A cylindrical capacitor has inner radius a and outer radius b and a length d (d>>b). The outer conductor is grounded and the inner is at a potential Vo. A uniform magnetic field B is applied parallel to the axis of the capacitor. The volume of the capacitor is filled with a plasma containing n free electrons per unit volume and an equal number of positive ions. The collisional relaxation time of the electrons is τ. You may neglect: i) the motion of the heavy ions, ii) the magnetic fields produced by the currents in the plasma, and iii) any fluctuations in the density of electrons or ions. Call

$$\omega_c = \frac{eB}{mc} \text{ and } \sigma_o = \frac{ne^2\tau}{m}$$

Calculate:

a) the electric field in the capacitor

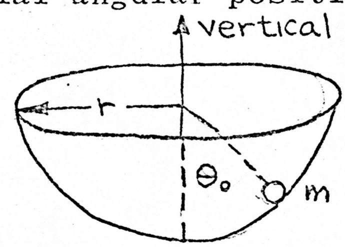
b) the current density in the capacitor

c) the resistance of the capacitor

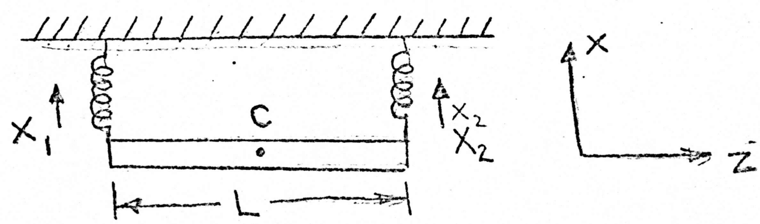
(Hint: look for steady state solutions.)

B. Do 2 problems.

B1. A particle is projected horizontally along the interior of a smooth hemispherical bowl of radius r which is kept at rest. We wish to find the initial speed v_o required for the particle to just reach the top of the bowl. Find v_o as a function of θ_o, the initial angular position of the particle.



B2. A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two massless springs attached one at each end.



The springs have the same force constant k. The motion of the center of gravity is constrained to move parallel to the vertical X axis. Find the normal modes and frequencies of vibration of the system, if the motion is constrained to the XZ-plane.

B3. A particle of mass m and total energy E is incident on a particle of mass M at rest. Find the maximum energy which the incident particle can transfer to the target, and the low velocity and extreme relativistic approximations to this expression.

C. Do 2 problems.

C1. Show that the difference between the heat capacity C_E of a dielectric for constant electric field E and the heat capacity C_D for constant displacement D (constant volume in both cases) is

$$C_E - C_D = \frac{TE^2}{4\pi\epsilon}\left(\frac{\partial\epsilon}{\partial T}\right)^2$$

Here use was made of the fact that $D = \epsilon(T)E$ and that the

first law of thermodynamics for a dielectric can be written

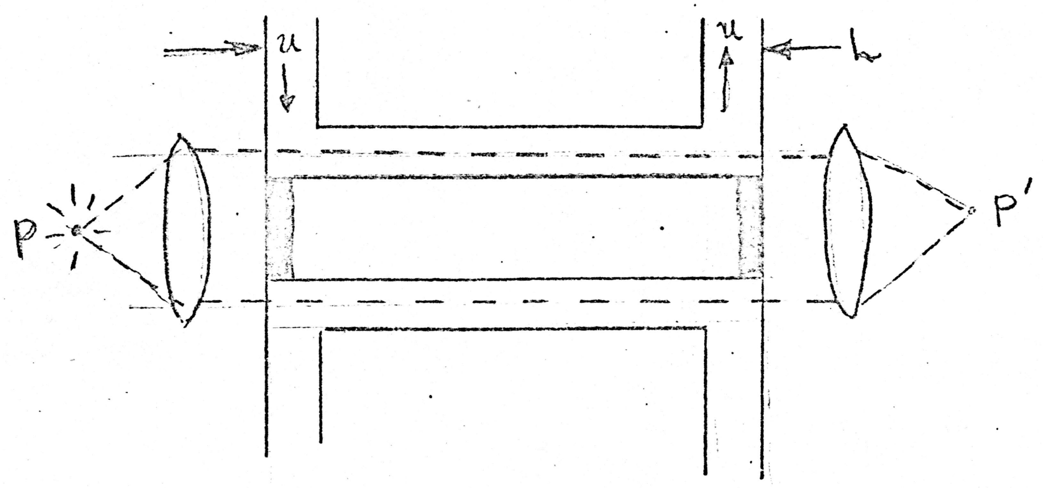
$$dQ = d\left(U + \frac{E^2}{8\pi}\right) - \frac{EdD}{4\pi}, \quad U = \text{internal energy}$$

C2. A solid contains N mutually noninteracting nuclei of spin 1. Each nucleus can therefore be in any of three quantum states labeled by the quantum number m, where $m=0, \pm 1$. Because of electric interactions with internal fields in the solid, a nucleus in the state $m=1$ or in the state $m = -1$ has the same energy $\epsilon>0$, while its energy in the state $m=0$ is zero.

Derive an expression for the entropy of the N nuclei as a function of the temperature T, and an expression for the heat capacity in the limit $\epsilon/kT \ll 1$.

(Hint: Calculate the free energy in terms of the partition function.)

C3. Light from a source of frequency ν is led through the system shown. The upper conduit carries a liquid having index of refraction n and moves with velocity u. The lower conduit contains the same liquid at rest. What is the minimum value of u that will cause destructive interference at P'?



I. MATH METHODS

Ph.D. Qualifying Exam
(Closed Book)

May 16, 1968
2 PM - 3:30 PM

This is $\frac{1}{2}$ of the examination for this afternoon and hence only ~ $1\frac{1}{2}$ hours should be spent on this section.

I. Work all four problems in this section.

A. Consider the symmetric matrix:

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$$

If $\lambda_i (i=1,2,3)$ are the eigenvalues of the matrix find:

$$\alpha = \sum_{i=1}^{3} \lambda_i$$

and $$\beta = \sum_{i=1}^{3} \lambda_i^2.$$

B. Making use of the residue theorem, show that

$$\int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x} d_x = \frac{\pi}{\sin a\pi}$$

C. 1) Show that the Lagrange function describing the small transverse motions around the equilibrium position of a homogeneous perfectly flexible string of length ℓ, linear mass density ρ, rotating around an axis with angular velocity ω is (in a plane coordinate system rotating with the string):

$$\mathcal{L}[u] = \int_0^t \int_0^{\ell} \left[\frac{\rho}{2}\left(\frac{\partial u}{\partial t}\right)^2 + \frac{\rho\omega^2}{2}\left(\ell^2 - x^2\right)\left(\frac{\partial u}{\partial X}\right)^2\right] dXdt.$$

Here u denotes the displacement of the string in a plane formed by the axis of rotation and the equilibrium

position (the tension is due to centrifugal force only; neglect gravity). The X=0 end of the string is fixed, the X=ℓ end is free.

2) Show that the Euler equation corresponding to the preceding Lagrange function is

$$\frac{\partial}{\partial X}\left((\ell^2 - X^2)\frac{\partial u}{\partial X}\right) - \frac{2}{\omega^2}\frac{\partial^2 U}{\partial t^2} = 0$$

with the boundary conditions:

$$U(0,t) = 0$$

$$U(\ell,t) = \text{finite.}$$

D. Solve the equation in C2) with the given boundary conditions and with initial conditions

$$U(X,0) = \alpha\left(\frac{X}{\ell}\right)^3 \qquad \alpha \text{ a small constant}$$

$$\frac{\partial U(X,0)}{\partial t} = 0$$

Hints: i) The Legendre differential equation

$$\frac{d}{dX}\left[(1-X^2)\frac{dy}{dX}\right] + \lambda y = 0$$

with boundary conditions $y(0) = 0$, $y(1) = $ finite has the eigenvalues

$$\lambda_K = (2K-1)(2K) \qquad K = 1,2,3,\ldots$$

and the normalized eigenfunctions

$$y_K(X) = \sqrt{\frac{4K-1}{2}}\, P_{2K-1}(X)$$

where $P_{2K-1}(X)$ are the ordinary Legendre polynomials

$$P_1(X) = X$$

$$P_3(X) = \frac{5}{2} X^3 - 3/2X$$

etc.

ii) The Legendre polynomials of odd degree form a complete system for the interval [0,1].

II. GENERAL PHYSICS

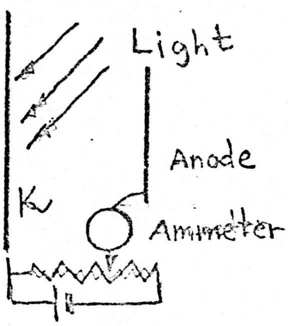
Ph.D. Qualifying Exam

May 16, 1968
3:30 - 5 PM

Section A

All questions to be answered. These are quick order of magnitude problems and if you cannot remember approximate values of constants etc., make a reasonable guess.

A-1. A surface of metallic potassium is illuminated with monochromatic of various wavelengths. In each case, the potential difference is found for which no electrons reach the anode. Two pieces of data are given:



Wavelength (Å)	2000	5000
Potential difference-volts	4.11	0.41

Find the value of Planck's constant.

A-2. The "saturation" field that can be obtained from magnetising iron is about 20,000 gauss. Assuming that this field is produced by the alignment of a few electron spins per atom, estimate the spin-magnetic moment of the electron.

A-3. The flux of energy from the sun at the earth is about 2 calories per square centimeter per minute on an area perpendicular to the direction of incidence. Under the assumption (a very poor one, to be sure) that the incident radiation is monochromatic, estimate the amplitude of both B and E.

A-4. Hydrogen in the ground state is in a $2s_{1/2}$ state. The spin of the proton is 1/2 and the hyperfine structure splitting is about 1420 Mh_z. The proton magnetic dipole moment is about 2.8 nuclear magnetons. Estimate the effective magnetic field within the atom at the point where the proton exists.

Section B

Answer two questions in Section B.

1) Describe the essential features of the phenomenon of superconductivity. You might include in your discussion topics such as the Meissner effect, Flux quantization and the Josephson Junction. (What does BCS stand for?)

2) Discuss carefully the principles and uses of lensless photography, i.e., Holography.

3) The laser is now solidly entrenched in everyday technology. Discuss both theoretically and experimentally the fundamentals of its operation.

4) Einstein's general theory of relativity is said to have been challenged by a recent measurement of solar oblateness. Elaborate.

5) A background radiation filling the universe has recently been measured. Discuss this measurement, its results, and its implication on cosmology.

6) Choose some topic which you consider to be of particular interest in the present stage of development of physics, and explain in 500-1000 words what you consider the interesting problems in this field, and how they are being investigated.

MODERN PHYSICS

Ph.D. Qualifying Exam
(Closed Book)

May 17, 1968
1PM to 4PM

Do 6 out of 7 of the following problems

1. Solve the harmonic oscillator problem for the ground state wave function and ground state energy by a variational calculation. A good trial function is $\psi = Ne^{-\alpha x^2}$

$$\left[\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1\cdot 3\cdot 5\cdots(2n^{-1})}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}\right]$$

2. At time t=0 an electron has its spin in the x direction. A magnetic field is present in the +z direction. What is the probability P_+ that the electron will be found with spin in the +x direction at time t.

3. Which of the following processes can occur physically? Make sure to explain your answers carefully.

   a) Emission of a photon by an electron moving in free space.

   b) Decay of a very energetic proton into a positive pi meson and a neutron.

   c) Decay of a spin zero excited state of a nucleus into the spin zero ground state by emission of a photon.

   d) Nuclear decay via α-particle emission between 0 angular momentum levels.

   e) $O^{14} \rightarrow N^{14} + e^+ + \nu$ (The O^{14} and N^{14} each have a 0^+ ground state)

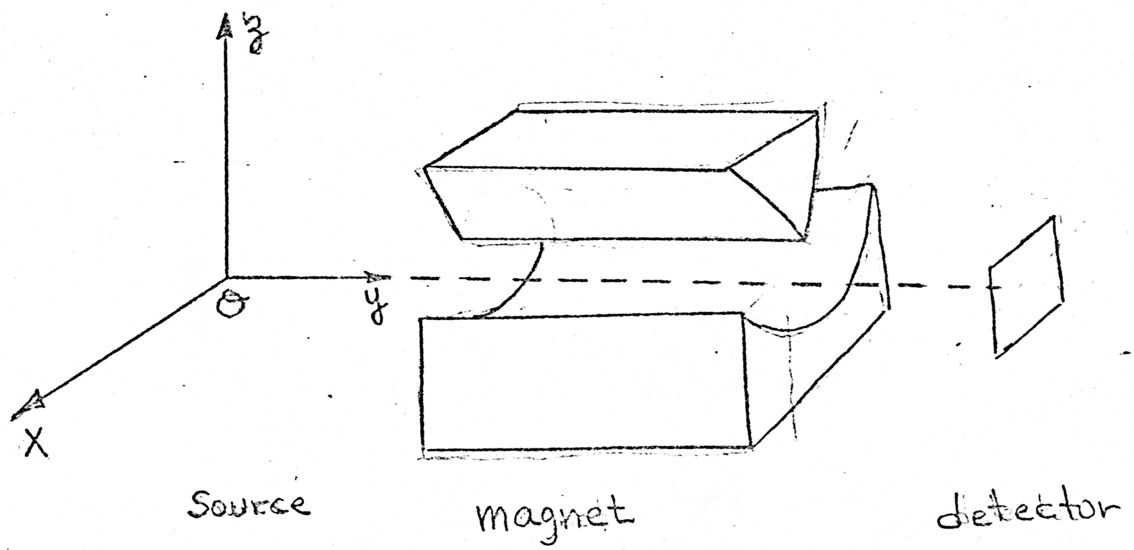
   f) $\mu^+ \rightarrow e^+ + \gamma$

MODERN PHYSICS (CONT)

4. a) What is the total angular momentum in the following states: ^1S, ^3S, ^3P, ^2D

   b) Which terms (states) can be realized for the following two electron systems: $ns\,n's$; $npn'p$; $(nd)^2$

5.



   a) Show, using the Heisenberg uncertainty relation, that a Stern-Gerlach apparatus (indicated above) is an impractical instrument with which to measure the magnetic moment of a free proton.

   b) Explain, if (a) is true, how this type of technique is able to measure magnetic moments.

6. Let $C_+ = \langle K^o | \psi \rangle$ and $C_- = \langle \bar{K}^o | \psi \rangle$ be the amplitudes to find the state ψ in one of the states K^o or $\bar{K}^o$ respectively at time t. To 1st order let K^o and $\bar{K}^o$ be such that

$$i\hbar \frac{dC_i}{dt} = E_o C_i \qquad i = +, -$$

However both K^o and $\bar{K}^o$ can decay into a π^+ and π^- meson via the weak interactions. That is the following weak couplings exist:

$$K^o \rightleftarrows \pi^+ + \pi^- \rightleftarrows \bar{K}^o$$

$$K^o \rightarrow \pi^+ + \pi^- \rightarrow K^o$$

$$\bar{K}^o \rightarrow \pi^+ + \pi^- \rightarrow \bar{K}^o$$

Because of the symmetry of matter and antimatter, the amplitude for all of these processes are equal, that is

$$\langle K^o|W|K^o\rangle = \langle \bar{K}^o|W|K^o\rangle = \langle K^o|W|\bar{K}^o\rangle = \langle \bar{K}^o|W|\bar{K}^o\rangle = A$$

Where W schematically indicates the weak interaction through the 2π state.

a) Show that if the weak 2π interaction is included the equations are

$$i\hbar\, dC_+/dt = E_o C_+ + A(C_+ + C_-)$$

$$i\hbar\, dC_-/dt = E_o C_- + A(C_+ + C_-)$$

b) Solve these equations [Hint: a standard notation for the new base states is $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K^o\rangle \pm |\bar{K}^o\rangle)$].

c) Suppose at t=0 there exists a pure K^o state. What is the probability that at time t this will act like a $\bar{K}^o$ state?

7. A "rigid" 2-dimensional rotator (moment of inertia I) carries an electric charge and hence a rotating dipole moment. (Neglect radiation effects.)

a) Find the Schrödinger equation for the stationary states of the rotator.

b) Solve for the energy levels and wave functions of the rotator.

Now turn on a weak uniform electric field $\underline{E}$ in the plane of the rotator.

c) What is the new Hamiltonian?

d) Find the new energies to the lowest order that produces a change.

e) What is the direction and net induced dipole moment of the n^{th} state of the rotator?

GENERAL PHYSICS - SHORT ANSWER
Answer 3 Questions

Ph.D Qualifying Exam   October 24-25, 1968

(1) Making use of the equivalence principle (gravitation equivalent to accellerated frame) derive an expression for the gravitational red shift. Is this measurable near the earths surface?

(2) Hydrogen gas in a laboratory discharge tube shows a discrete spectrum. At the same temperature and pressure, but in a massive system such as a star, it shows more nearly a black body radiation spectrum. Explain.

(3) Can a spin-$\frac{1}{2}$ system have a permanent electric dipole moment if

(a) Parity is conserved

(b) Parity is not conserved but time reversal invariance holds

Why?

(4) Can a gamma ray convert into an electron-positron pair in (i) free space (ii) an aluminum plate (iii) a lead plate? In which medium is the mean free path for conversion shorter?

(5) The decay of a ρ^o meson (spin-parity = 1^-)

$\rho^o \to \pi^+\pi^-$ is allowed, but

$\rho^o \to \pi^o\pi^o$ is not

Why?

(6) Can a Σ^+ hyperon (spin-parity $\frac{1}{2}^+$) have a static electric quadrupole moment? Why.

(7) If a medium has a non-linear characteristic such that the velocity of propagation of a pulse increases with the amplitude of the pulse, what will happen to a disturbance for which the amplitude as a function of position is bell shaped initially?

GENERAL PHYSICS - SHORT ANSWER

GENERAL PHYSICS - SHORT ANSWER

(8) (a) What is the total angular momentum in the following states?

$^{1}{}_{s}$, $^{3}{}_{s}$, $^{3}{}_{p}$, $^{2}{}_{d}$

(b) Which states can be realized for the following two electron systems?

(NS, N′S); (NP, N′P); Nd, Nd;

THERMODYNAMICS

THERMODYNAMICS

Ph.D Qualifying Exam October 24-25, 1968

Consider a rubber band with equation of state

$$F = KT(L-Lo)$$

over the physical region of interest. Here F is the tension, K a constant, T the temperature, L the length and Lo(T) the unstretched length. If you suddenly stretch a rubber band and place it to your lip you will find that it is warm.

(a) The rubber band is under tension by a suspended weight and in thermal equilbrium in a refrigerator. Will weight rise or fall when the band and weight are brought into a warm room.

The band now undergoes an isothermal process, calculate

(b) the change in internal energy and

(c) the heat transfered.

(d) Describe in principle how you could construct a cyclically operating refrigerator using this rubber band as a working substance (You may assume an ideal incompressible fluid is available for the heat transfers.)

Ph.D Qualifying Exam October 24-25, 1968

Consider the following simple model for paramagnetic behavior in a gas or crystal. There are n magnetic ions per unit volume each with dipole moment $\underline{M}$ and angular momentium $\frac{1}{2}$. When the substance is placed in a magnetic field the ions will tend to allign themselves in the direction of the applied field since the potential energy of an individual dipole is $U = -\underline{M}\cdot\underline{B}$. The tendency toward alignment is opposed by disorienting effects of thermal agitation. Calculate the specific heat and the paramagnetic succeptibility

(a) Quantum mechanically

CLASSICAL

(b) By assuming that the ions behave as classical dipoles.

(c) Discuss qualitatively the calculation of the susceptibility in a metal, using a free electron model.

CLASSICAL MECHANICS

Ph.D Qualifying Exam October 24-25, 1968

Consider a bead of mass m attached to the middle of an elastic string of length L. The tension in the string is T, . The string is mounted vertically at the center of a turntable, that rotates with an angular velocity ω. A rod through the center of the bead constrains it to move in a fixed plane relative to the turntable.

(a) Write down a Lagrangian describing the motion of the bead. Use a coordinate system fixed to the turntable. It may be assumed that the displacement of the bead is small and only in the horizontal direction.

(b) What is the frequency of oscillation of the bead?

(c) What happens when $\omega > \left(\frac{T}{mL}\right)^{\frac{1}{2}}$ $\omega > \left(\frac{2T}{mL}\right)^{1/2}$

(d) The rod is now removed. What is the Lagrangian that describes the system? It may be assumed that the displacements are still only horizontal.

(e) Are there any conserved quantities?

(f) If the bead is displaced from the center by someone standing on the turntable, and then released, will it ever pass over the center of the turntable?

CLASSICAL MECHANICS

Ph.D. Qualifying Exam October 24-25, 1968

A charged particle of charge q and mass m is ;at rest at the origin of the coordinate system at time t=0. There is a static, spatially uniform magnetic field B present parallel to the z axis. For times t>0, the particle is perturbed by an electric field $\underset{\sim}{E}=xE_o\cos(\Omega t)$. Solve the resulting equations of motion. Discuss the nature of the motion for the two cases $\Omega \neq \Omega_c$, and $\Omega \equiv \Omega_c$, where $\Omega_c=(\frac{qB}{mc})$.

MATHEMATICAL PHYSICS

(Answer All Questions)

Ph.D Qualifying Exam — October 24-25, 1968

(1) (a) Suppose that A, B and C are Hermitian operators and that

$$AB - BA = \alpha' C$$

where α' is a complex number, show that α' is purely imaginary.

(b) Consider the equation $\nabla^2\varphi - \lambda\varphi = 0$, $\lambda>0$, with the condition that $\varphi = 0$ on a closed smooth surface. Show that this equation has no non-trivial solutions inside the surface by proving that

$$\int_V \left(\nabla^2\varphi\right) \varphi dv < 0$$

V is the volume enclosed by the surface. How does this imply the stated result?

(c) Show that a charged particle in an otherwise charge free region of space cannot be in stable equilibrium if acted upon by electrostatic forces. You may simply state the relevant mathematical theorem.

(2) Suppose that the velocity of light in a medium varies linearly in the Z direction. Light rays that travel at an angle to the Z axis will be refracted, and will not travel in straight lines. Using Fermat's principle [The path between two points followed by a light wave is such as to minimize the time of travel] show that the paths (in the xz plane) in such a medium are circles.

Hint: Use distance in the x direction as the independent variable. Since the velocity doesn't depend upon x, x it is possible to obtain a first integral of the equations for the path.

(3) Evaluate (a) $$\int_0^\infty \frac{\cos x \, dx}{x^2 + a^2}$$

x^2

(b) $$\int_0^\infty \left(1 + x^2\right)^{-2} \log x \, dx$$

(4) Consider the integral equation

$$\lambda e^{\lambda t} f(t) - \int_0^\infty \cos t \cos t' \, f(t') dt' = \sin t \qquad \lambda > 0$$

Find the solution $f(t)$ for a $\lambda \ll 1$
b $\lambda \gg 1$

(c) Are there any values of $\lambda > 0$ for which a solution does not exist?

QUANTUM MECHANICS

QUANTUM MECHANICS

Answer All Questions

Ph.D Qualifying Exam

October 24-25, 1968

Time Allotted: 3 Hrs.

(1) Two identical bosons move in a one-dimensional harmonic oscillator potential. In addition they interact with each other with a potential

$$V(x_1, x_2) = \alpha e^{-\beta(x_1 - x_2)^2}$$

where β is a real positive constant. Find the ground state energy of the system to lowest order in the interaction strength parameter α.

(2) (a) Use the Born Approximation to find the differential cross-section for scattering of a spin-less particle of wave vector $\vec{k}$ from a potential

$$V(r) = \alpha e^{-\beta r}$$

where α, β are real constants.

(b) For what range of energy would this approximation be valid?

(c) Indicate how you may evaluate the phase shift for the $\ell^{\underline{th}}$ partial wave as a function of energy in the region for which the result of (a) is valid.

(3) A tritium atom (H^3) is in its ground state. The triton undergoes a β-decay leaving behind a singly ionized He^3 atom. What is the probability that the final atom is in its ground state? Assume for simplicity that both nuclei are infinitely massive and that the β-decay electron has no interaction with the rest of the system. Justify any other assumptions or approximations made in working the problem. How, in principle, might your result be experimentally tested?

(4) (a) Consider two spin-$\frac{1}{2}$ systems bound together in a manner approximating a rigid diatomic molecule. Assume there is a relatively weak spin-orbit coupling of the form

$a\,\vec{L}\cdot\vec{S}$, where $\vec{L}$ = orbital angular momentum, $\vec{S}$ = total

spin of the system, and $a \ll \frac{1}{I}$ where I is the moment of

inertia of the dumbbell.

Denoting the states by the usual spectroscopic notation, describe the gross structure and fine structure of the levels of the system.

(b) Assume that in addition there is a very weak spin-spin interaction of the form

$$b\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

where $b \ll a$. What is the primary extra feature introduced into the level structure by this term?

(c) Assume, that the spin-orbit and spin-spin interaction arise from the electromagnetic properties of the two spin-$\frac{1}{2}$ systems. What are the relevant electromagnetic parameters, and what must the relative magnitude of these parameters be in order to produce interactions of the relative sizes described in a and b.

ELECTRICITY AND MAGNETISM

Answer All Questions

Ph.D Qualifying Exam — October 24-25, 1968

Time Allotted: 3 Hrs.

(1) A point charge q is located a distance d above an infinite conducting plane. Find

(a) The surface charge density on the plane (as a function of position on the plane).

(b) The force between the plane and the charge.

(c) The work done on the charge in bringing it from infinity.

(d) The potential of q with respect to the plane.

(2) A stationary charged particle is located inside a long solenoid, a distance r from the axis. The field is switched on suddenly. Show that the particle executes a circular orbit of radius r/2, about a point a distance r/2 from the axis.

(3) Show that a sphere whose charge density is spherically symmetric, and that oscillates in a purely radial mode will not radiate.

(4) A plane polarized E.M wave traveling in a vacuum is incident upon a dielectric slab of permittivity ϵ, and thickness d. Find ϵ and d so that there is no reflected wave.

GENERAL PHYSICS - ESSAY ~~SHORT ANSWER~~ ESSAY (ANSWER ONE)

(Answer One Question)

Ph.D Qualifying Exam October 24-25, 1968

(1) Discuss generally the type of spectrometers that might be used to investigate the following:

(a) Spectrum of light emitted by a faint star

(b) Hyperfine structure of atomic transitions

(c) Mode structure of a laser

Describe one optical system in detail. Indicate the criteria you have used to make your choice.

(2) The lifetimes of atoms in excited states is commonly about 10^{-8} sec. How would you verify the correctness of this statement? With what precision can the lifetime be defined and/or measured? Indicate what you consider to be the major difficulty in such an experiment.

(3) Discuss within the framework of quantum mechanics the relation between symmetry principles and conservation laws. Give examples of at least three continuous transformations (not all involving space time) and two discrete transformations. Identify the associated conserved quantities. Discuss the experimental validity of the conservation laws.

(4) Many of the properties of solids can be understood in terms of the concept of an elementary excitation. Examples of such excitations are phonons, plasmons, magnons, helicons, polaritons, rotons. Choose three of these excitations, (or any others that you can think of) and describe them in some detail, including the kind of system they exist in, the means by which one might excite and observe them, and indicate qualitatively what their dispersion relation is. For at least one of these, describe in detail the changes in physical quantities that accompany the excitation, and give order of magnitude estimates of the parameters that characterize this excitation in a real system. Indicate any restrictions on wave vector, frequency or other physical parameters that must be observed if this excitation is to be well defined.

(5) Describe the essential features of the phenomenon of superconductivity. Include in your discussion topics such as the Meissner effect, flux quantization and the Josephson junction. Describe at least two distinct experiments by which the relation $J=J_0\sin(\varphi)$, where J is the superconducting current through the junction and φ the phase difference in the order parameter, can be checked.

(6) A background radiation filling the universe has recently been measured. Discuss this measurement and its implications for cosmology.

PART I

Ph.D. Qualifying Exam
Spring 1969

Thursday, May 15, 1969
9 AM - 12 NOON

Budget your time to answer all 7 questions.

1. Write the Lagrangian of a Foucault pendulum. Derive an equation for the angular velocity of the plane of a Foucault pendulum as a function of latitude.

2. The neutral pi meson decays in flight into two photons. Find the angular distribution of the photons.

3. An electric dipole of moment $\overline{p}$ is at the center of a spherical dielectric shell with $\kappa\epsilon_o$ and inner and outer radii a and b. Find the potential everywhere. Describe briefly how one would find the induced charge densities on the two surfaces.

4. Find an expression for the fields of the lowest order transverse-electric field wave in a rectangular conducting pipe (width = a, height = b). This fundamental wave is defined to be that for which there is no field variation in the "height" direction. Sketch the dispersion diagram for this wave (ω vs. k) and show by construction how you would find the resonant frequencies in this mode if a cavity were formed by the insertion of two shorting planes a distance ℓ apart in the guide.

5. Using the Lorentz invariance of phase, derive the relativistic Doppler shift expression for a plane wave traveling in an arbitrary direction with respect to a moving frame.

   Now consider a system which is capable of emitting photons of frequency ν at an equal rate in opposite directions to infinity. At the time that 2n photons have been emitted, the transmitter is turned off. Calculate the total photon energy in a frame in which the transmitter is at rest and in a laboratory system in which it is moving at a velocity v. By ascribing the laboratory result to be due to kinetic energy of the photons, derive an approximate ($v \ll c$) expression for the mass of the photon assembly.

6. A dilute gas is described by the van der Waals equation of state

$$(p + a/V^2)(V - b) = RT$$

with small positive a, b. Compute:

1) The entropy;
2) The Helmholtz free energy;
3) $C_p - C_V$, up to quantities of second order in a, b;
4) The work done by one mole of gas in a reversible isothermal expansion.

7. Part c) of this problem depends on solving part a), b).

$\hbar\omega$

a) Find the classical and quantum-mechanical partition function of a system of identical harmonic oscillators and show that the first is the limit of the second for $\hbar\omega/kT \to 0$.

b) Determine the internal energy and the heat capacity of such a system.

c) Consider the small vibrations of the N atoms making up a crystal lattice as a system of harmonic oscillators. Debye has proposed to approximate the normal modes of such a system by the elastic waves in a continuum (there are two types of such waves, longitudinal and transverse, the latter with two polarizations, propagating with velocities c_ℓ and c_t, respectively). Assuming that the number of vibrational modes with frequencies between ω and $\omega + d\omega$ has the form $g(\omega)\,d\omega$, with: (Can you justify this formula?)

$$g(\omega) = \begin{cases} (V/2\pi^2)\left(\dfrac{1}{c_\ell^3} + \dfrac{2}{c_t^3}\right)\omega^2 \equiv (9N\omega^2/\omega_D^3) ; & \omega \;\; \omega_D \\ 0 ; & \omega < \omega_D \end{cases}$$

where ω_D is the Debye frequency, derive from this model and the results of the preceding parts of the problem the heat capacity of a solid and discuss the high- and low-temperature limits. Can you explain the qualitative meaning of the Debye frequency?

Part II

Ph.D. Qualifying Exam
Spring 1969

Thursday, May 15, 1969
2:00 - 5:00 PM

Part II consists of two sections: Mathematical Methods and General Questions. Try to spend approximately equal time on each.

6

Mathematical Methods

1. Show that

$$PV \int_0^\infty \frac{x^{a-1}}{1-x}\, dx = \pi \cot(a\pi) \qquad (0 < a < 1)$$

where PV denotes the principal value of the integral.

Hint: The integrand is a branch of a many-valued function, which can be made single-valued by cutting the complex plane along the positive real axis.

2. In a coordinate system with the base-vector e_1, e_2, e_3; e_4 of a four-dimensional space a linear operator has the matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 2 \\ -1 & 0 & 3 & 1 \\ 2 & 1 & 5 & -1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

What will be the form of the matrix representing the operator M in the following two new bases:

1) e_1, e_3, e_2, e_4;

2) $e_1, e_1 + e_2, e_1 + e_2 + e_3, e_1 + e_2 + e_3 + e_4$

3. Solve the heat equation

$$u_t - a^2 u_{xx} = 0$$

with the boundary conditions:

$$u(0,t) = 0$$

$$u(\ell,t) = bt, \quad t > 0$$

and the initial condition

$$u(x,0) = \sin \frac{3\pi}{\ell} x.$$

Hint: Since the boundary condition $u(\ell,t) = bt$ is inhomogeneous, corresponding to the heating of one end of the bar, special care is needed to treat this problem; one possibility is to find a simple solution of the equation (e.g. a polynomial in x and t) which satisfies the equation and both boundary conditions, and to subtract this from the unknown function.

General Questions

1. Which of the following conservation laws is well established, approximate, or merely conjectured? For those where experimental evidence exists, to what accuracy is the law established?

a) Energy
b) Electric charge
c) Parity and CP
d) Time reversal
e) Isospin
f) Strangeness or hypercharge
g) Helicity
h) Baryon number
i) Lepton number
j) Muon number

2. Last year's Nobel prizes in Physics and Chemistry were awarded to Luis Alvarez and Lars Onsager, respectively, both for contributions to Physics. What were these contributions?

3. Explain very briefly the fundamental aspects of ferromagnetism and antiferromagnetism (not more than ½ page).

4. What is the approximate angular resolving power of a telescope aperture?

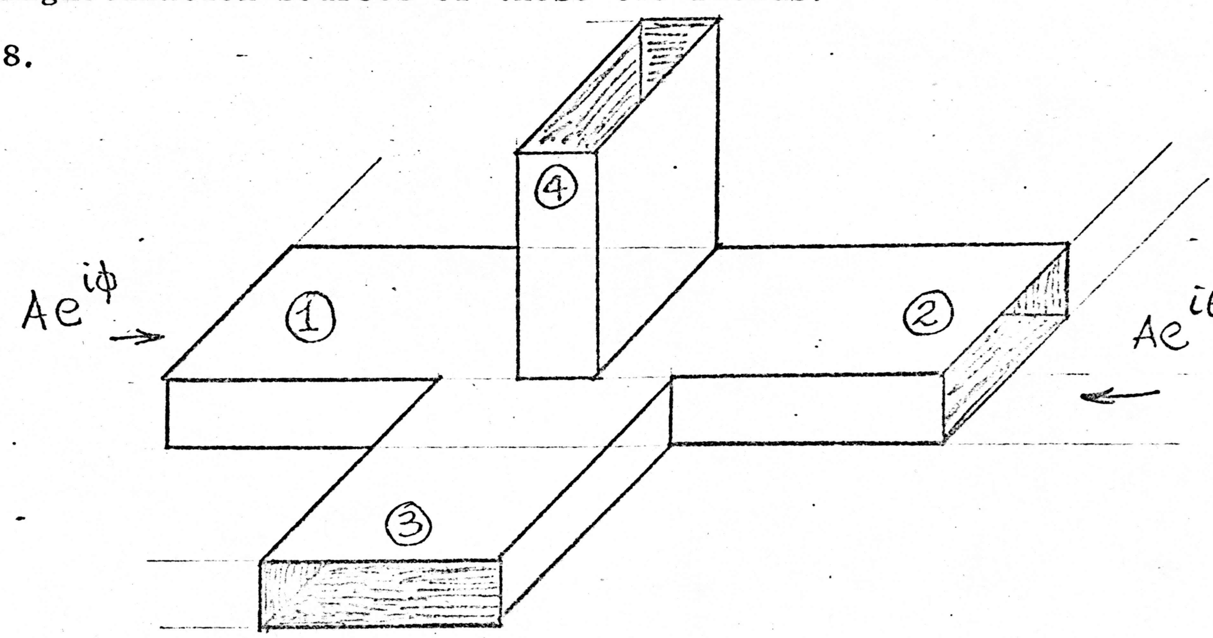
5. Can a "race course" be found in which an electron may win a race with a photon? Explain.

6. What has the following expression to do with the charge of the electron?

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

7. Consider an iron cylinder with uniform magnetization along its axis. If the length is approximately equal to the diameter, sketch the B and H fields everywhere. Give expressions for the magnetization sources of these two fields.

8.



Two waves in the lowest TE mode, with equal amplitude and phases φ and θ, enter ports 1 & 2 of the junction shown. If there are no reflections, what signals appear from ports 3 & 4. Explain.

9. Why do the tides on the earth have a twelve-hour period rather than a twenty-four hour period?

10. What two recent observations have revolutionized our concepts about pulsars?

11. Why do hurricanes in the Caribbean circulate in an anticlockwise sense?

12. Pick one colloquium (on a physics topic) and describe, in a short paragraph, the principal concepts involved and the results obtained.

13. Describe, in a short paragraph, the subject and main points of your talk this year before the Graduate Student Seminar.

Ph.D. Qualifying Exam
Spring 1969

PART III

Friday, May 16
1 - 4 p.m.

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1. Let P be the parity operator, defined by the property that it changes the sign of each coordinate and each momentum component, or by ($\vec{q},\vec{p}$ denote the coordinate and momentum operators)

$$P\vec{q} + \vec{q}P = 0, \quad P\vec{p} + \vec{p}P = 0.$$

Show that $P^2 = 1$, and that P commutes with the orbital angular momentum operator $\vec{L} = \vec{q} \times \vec{p}$. Denoting by ψ_ℓ a simultaneous eigenfunction of L^2 and P show that the eigenvalue of P corresponding to the eigenvalue $\ell(\ell+1)$ of L^2 is $\pm(-1)^\ell$.

2. Consider two linear harmonic oscillators of the same frequency and equilibrium position (both move along the x-axis) which are coupled by a force proportional to their mutual distance.

Solve the Schrödinger equation for this system and determine the energy levels corresponding to symmetric and antisymmetric states (label the energy eigenvalues by their quantum numbers, including parity).

3. The S-wave interaction of a proton and a neutron (both spin $\frac{1}{2}$) can be described in a good approximation by two central forces, different for the singlet and the triplet states of the system. Express this circumstance in terms of a single spin-dependent potential, using either a "spin-exchange operator" (Σ_{np}, acting on the spin eigenfunctions by $\Sigma_{np}|s_p, s_n\rangle = |s_n, s_p\rangle$) or directly in terms of the spin operators of the two particles.

Correlate your result with what you know about the empirical properties of the deuteron.

Ph.D. Qualifying Exam
Spring, 1969

PART III

Friday, May 16
1 - 4 p.m.

4. Compute the first-and second-order perturbations to the energy eigenvalues of a two-dimensional rotator subjected to a weak uniform electric field along the x-axis. (A two-dimensional rotator is a mass point moving in the plane at a fixed distance from the origin; for this problem, assume that the charge of the mass point is e. Use the coordinate system in which the Schrödinger equation and the boundary condition for it are simplest).

5. Calculate the first Born approximation for the scattering cross section of a particle on a "screened Coulomb potential"

$$V(r) = (Ze^2/r)\ \exp(-ar)$$

and show that in the limit $a \to 0$ you obtain the Rutherford formula.

6. What is the difference between hyperfine structure and Lamb shift? Draw term diagrams.

7. What is the general relation between mean life for alpha decay and the energy of the emitted alpha particle? Formulate briefly the basis of the quantum mechanical explanation of this relation.

PART I - CLASSICAL PHYSICS

Ph.D. Qualifying Exam
Fall, 1969

Thursday, Sept. 25, 1969
1 - 5 PM

There are twelve problems in this section. You are to attempt nine of the twelve. The problems that are required are indicated, as well as the optional ones. You have enough time to spend an average of 25 minutes per problem.

Useful Mathematical Formulae

$$\frac{1}{|\underset{\sim}{r} - \underset{\sim}{r}'|} = \sum_{\ell} \frac{r_<^{\ell}}{r_>^{\ell+1}} P_\ell(\hat{r} \cdot \hat{r}')$$

$$P_\ell(\hat{r} \cdot \hat{r}') = \frac{4\pi}{2\ell + 1} \sum_m Y^*_{\ell m}(\hat{r}) Y_{\ell m}(\hat{r}')$$

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{1,0} = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \cos\theta$$

$$Y_{1,\pm1} = -\left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \sin\theta \, e^{\pm i\varphi}$$

$$Y_{2,0} = \left(\frac{5}{4\pi}\right)^{\frac{1}{2}} \left(\frac{3}{2} \cos^2\theta - \frac{1}{2}\right)$$

$$Y_{2,\pm1} = -\left(\frac{15}{8\pi}\right)^{\frac{1}{2}} \sin\theta \cos\theta \, e^{\pm i\varphi}$$

$$Y_{2,\pm2} = \frac{1}{4}\left(\frac{15}{2\pi}\right)^{\frac{1}{2}} \sin^2\theta \, e^{\pm i2\varphi}$$

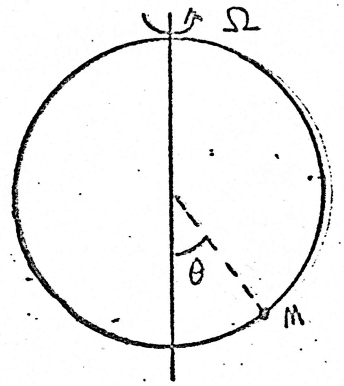
PROBLEMS 1 THROUGH 3 ARE REQUIRED

1. A billiard ball is struck by a cue in such a manner that the line of action of the applied impulse is horizontal, and (possibly) off center only in the vertical direction. The initial velocity V_o, angular velocity ω_o of the ball, its radius R and mass M are all known, as well as the coefficient of friction μ between the ball and the table.

   (a) How far will the ball move before a pure rolling motion begins?

   (b) Discuss the path followed by the ball while slipping after it is struck by a general cue stroke, i.e. one in which the impulse is not necessarily horizontal, nor off center only in the vertical direction.

2. Consider a bead free to slide on a circular wire of radius a. The wire is placed in a gravitational field of strength g, and is rotated about an axis parallel to the gravitational field with angular velocity Ω, as indicated in the sketch below:



   (a) Express the Lagrangian of the mass in terms of the angle θ. Find the equation of motion of θ.

   (b) Deduce the equilibrium position of the mass as a function of Ω. Be sure to consider the two cases $\Omega < (g/a)^{\frac{1}{2}}$ and $\Omega > (g/a)^{\frac{1}{2}}$. Sketch the equilibrium angle θ_o as a function of Ω.

   (c) Consider small oscillations about the equilibrium angle θ_o. Sketch the frequency ω of such small oscillations as a function of Ω for the two regimes $\Omega < (g/a)^{\frac{1}{2}}$ and $\Omega > (g/a)^{\frac{1}{2}}$.

3. Show that the equation of motion for a rocket projected vertically upward in a uniform gravitational field, neglecting atmospheric resistance, is

$$m \frac{dv}{dt} = -v' \frac{dm}{dt} - mg$$

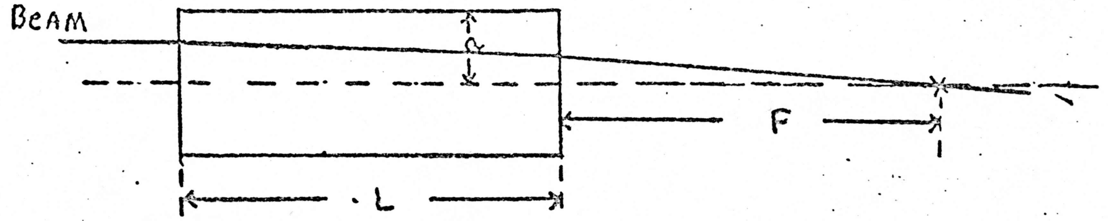
where m is the mass of the rocket and v' is the velocity of the escaping gases relative to the rocket ($v' > 0$). Assume a constant time rate of loss of mass and find v as a function of m. Let the rocket start initially from rest with $v' = 6800$ ft/sec with a constant mass loss per second equal to 1/60th of the initial mass (values appropriate to the V-2). Show that in order to reach escape velocity, the ratio of the weight of the fuel to the weight of the empty rocket must be almost 300.

<u>DO EITHER PROBLEM 4 OR PROBLEM 5</u>

4. Consider a cylindrically symmetric tube of constant current density a. A beam of particles of charge q enter this device. The beam is displaced from the axis of the tube, as shown in the sketch below.

   (a) Consider the "thin lens limit", as indicated in the sketch, and find an expression for the focal length. Express the result in terms of the momentum of the particles in the beam, the magnetic field B_o at the outer edge of the current column, and the quantities a and L indicated in the sketch.

   (b) State what you feel is the criterion for the validity of the thin lens approximation.



5. The betatron is a device utilized to accelerate charged particles.

   (a) Explain the operation of the betatron.

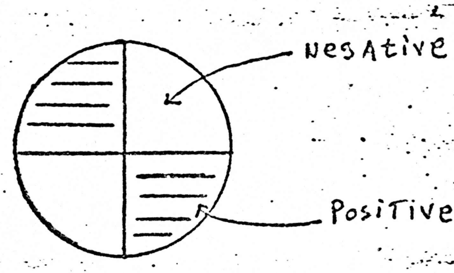
(b) Show that the equilibrium orbit radius of a particle will not change during the acceleration process if

$$\frac{dB}{dt} = \frac{1}{2}\frac{d\bar{B}}{dt},$$

where B is the value of the field at the orbit, and $\bar{B}$ is the average field inside the orbit.

DO EITHER PROBLEM 6 OR PROBLEM 7

6. Consider a thin, spherical shell charged in the wedgelike fashion indicated in the sketch below:



i.e. imagine the sphere constructed from four uniformly charged wedges, so that the total charge on the sphere is zero. Let the charge of each wedge be Q. Calculate the form of the electrostatic potential in the region $r \gg a$; where a is the radius of the sphere, and r measures the distance from the center of the sphere.

Required for the Solution:

$$\frac{1}{|\underset{\sim}{r} - \underset{\sim}{r}'|} = \sum_{\ell} \frac{r_<^{\ell}}{r_>^{\ell+1}} P_\ell(\hat{r}\cdot\hat{r}')$$

$$P_\ell(\hat{r}\cdot\hat{r}') = \frac{4\pi}{2\ell+1} \sum_m Y^*_{\ell m}(\hat{r})Y_{\ell m}(\hat{r}')$$

plus spherical harmonics of order 2.

7. A sphere of radius a carries a uniform charge distribution of density σ on its surface. The sphere is rotated about a diameter with constant angular velocity $\omega \ll c/a$. Find the magnetic field inside and outside the sphere.

PROBLEMS 8, 9 AND 10 ARE REQUIRED

8. Consider light in a medium with refractive index n_1. The light is incident on a plane surface separating the medium from another dielectric with refractive index $n_2 < n_1$.

(a) Find the angle of incidence θ_c such that total internal reflection is obtained for $\theta > \theta_c$. [θ = angle between the normal to the surface, and the direction of propagation.]

(b) For $\theta > \theta_c$, there is still a wave that extends into the rare medium. Show for incident radiation with electric field perpendicular to the plane of polarization that there is no energy flow into the rare medium, so all the energy is reflected.

9. A classical van der Waal's gas is described by the equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

(a) Show that if S_o is the entropy of the gas at the reference temperature T_o and reference volume V_o, then

$$S - S_o = C_V \ln\left(\frac{T}{T_o}\right) + R \ln\left[\frac{V - b}{V_o - b}\right].$$

Assume that C_V is independent of temperature in the derivation.

(b) Show that

$$C_p - C_V = R\left[1 - \frac{2a(V - b)^2}{RTV^3}\right]^{-1},$$

where C_p and C_V are the specific heat at constant pressure and constant volume respectively.

10. A vertical cylinder contains ν modes of a monatomic ideal gas and is closed off by a piston of mass M and area A. The whole system is thermally insulated. The downward acceleration due to gravity is g. Initially the piston is clamped in position so that the gas has a volume V_o and absolute temperature T_o. The piston is now released and, after some oscillation, comes to rest in a final equilibrium position corresponding to some smaller volume V of the gas where it has a temperature T. Neglect any frictional forces and neglect the heat capacities of the piston and cylinder. What is the final mean pressure P, T and V of the gas?

DO EITHER PROBLEM 11 OR PROBLEM 12

11. A circular tube of radius R is placed in a magnetic field H, inclined at an angle θ with respect to the ring, as shown below. The tube carries a current I by virtue of the motion of N particles of mass M and charge Q that circulate inside the tube. Each particle moves with velocity v in a circular path. (Think of the tube as a metal loop.)

(a) Calculate the torque excited on the ring, and indicate its direction.

(b) Discuss the motion of the ring, if it is allowed to move freely under the torque computed in part (a).

12. Consider the thermodynamic properties of a rubber band. Let the length of the rubber band be L, and let F be the tension of the band.

(a) Show that $(\partial S/\partial F)_T = (\partial L/\partial T)_F$.

(b) X-ray studies show that an unstretched rubber band has a more or less amorphous structure, while a rubber band that has been very slowly stretched is made up of oriented long molecules (i.e. it has a sort of crystalline structure). Suppose a rubber band is stored in a refrigerator under tension with a small weight attached to it. The band is then removed from the refrigerator and brought into a (warm) room. Does the length of the band increase or decrease? Be sure to discuss your reason for arriving at your conclusion.

PART II - MODERN PHYSICS

Ph.D. Qualifying Exam
Fall 1969

Friday, September 26, 1969
1 - 5 p.m.

There are nine problems in this section. All nine problems should be attempted. As in the first part of the exam, you should have an average of 25 minutes per problem.

Useful Formulas

$$S_+ \, |M\rangle = \left\{S(S+1) - M(M+1)\right\}^{\frac{1}{2}} \, |M+1\rangle$$

$$S_- \, |M\rangle = \left\{S(S+1) - M(M-1)\right\}^{\frac{1}{2}} \, |M-1\rangle$$

$$\psi_{1s} = \frac{1}{\sqrt{\pi}} \left(\frac{z}{a_o}\right)^{\frac{3}{2}} e^{-\frac{zr}{a_o}}$$

$$E_{1s} = -\frac{mz^2e^4}{2\hbar^2}$$

$$\iint dV_1 dV_2 \, \frac{e^{\frac{-2z}{a_o}(r_1+r_2)}}{r_{12}} = \frac{5\pi^2}{8z^5} \, a_o^5$$

1. Consider a square well of radius a with depth of potential V (V>o) which describes the interaction of two types of particles of equal mass M. If one type of particle, initially at rest, is bombarded by a beam of particles of the other type with energy 2E, show that the scattering phase shift δ for the S-wave is given by

$$k \cot \delta = \frac{\sqrt{k^2+K^2} + k \tan ka \tan \sqrt{k^2+K^2}\, a}{\tan \sqrt{k^2+K^2}\, a - \sqrt{1+\frac{K^2}{k^2}} \tan ka}$$

when $k^2 = ME/\hbar^2$, $K^2 = MV/\hbar^2$. What is the total cross section if only S waves are present? How can you predict which partial waves are important for a given a and E? Experimentally, how would you know if partial waves with ℓ>o are important?

2. Write down the Schrödinger equation for a particle in a general electromagnetic field. Show that physically significant quantities remain invariant under a gauge transformation $\underset{\sim}{A} \to \underset{\sim}{A}' - \nabla f$, and $\varphi \to \varphi' + \frac{1}{c}\frac{\partial f}{\partial t}$.

3. Consider a particle in one dimension, moving under the influence of the attractive potential

$$V(x) = -\lambda\left[\delta(x-\tfrac{a}{2}) + \delta(x+\tfrac{a}{2})\right], \quad \lambda > 0.$$

Discuss the bound states of a particle in this potential. In particular, derive the equations whose solutions yield the energies of the bound states (the plural is used in case there is more than one). Find an approximate analytic expression for the bound state energies valid in the limit $a \to \infty$. Also discuss the limit $a \to 0$.

4. The energy levels of a magnetic ion of spin S placed in a solid, in the presence of a magnetic field H, may often be described by the Hamiltonian (in the appropriate units)

$$\mathcal{H} = DS_x^2 - HS_z\ .$$

a. Find the energy levels of the system when $S = 1$, and make a level diagram that shows the field dependence of each level for this case.

b. Find the eigenfunctions when $S = 1$.

c. Suppose a time dependent magnetic field is applied parallel to the z axis. Indicate on the level diagram the transitions that may be induced by such a field. Make a sketch of the transition probability as a function of the strength H of the d.c. magnetic field of parts (a) and (b). (It should not be necessary to perform a detailed calculation to answer this part of the problem. You should be able to make a rough sketch by examining the results of (b).

5. Estimate the ionization energy of a normal helium atom, including the lowest effects of the electron-electron interactions.

6. Consider a system of two identical particles of spin $\frac{1}{2}$.

   a. What are the possible spin states of the system?

   b. What are the symmetry properties of the spin states under exchange of the spin coordinates of the two particles?

   c. Deduce the expression for the differential scattering cross section for collisions between these particles in terms of the scattering amplitude $f(\theta)$ for distinguishable particles having the same mass and interacting via the same force law. Assume no spin dependent or spin orbit forces and average over initial and final spins.

   d. What is the relationship at $\theta = \pi/2$ between the scattering cross section for identical particles and the cross section for distinguishable particles.

7. Consider the hydrogen-like system composed of an electron and a positron. The hyperfine coupling between the electron and the positron gives rise to a term in the Hamiltonian of the form $A\, \underset{\sim}{S}_p \cdot \underset{\sim}{S}_e$, where $\underset{\sim}{S}_p$ and $\underset{\sim}{S}_e$ refer to the spin of the positron and the electron, respectively. Discuss the energy levels associated with the orbital ground state in the presence of a magnetic field. Draw an energy level diagram and label the levels with the appropriate quantum numbers, where possible. Be sure to indicate the behavior in the two extreme limits, where the Zeeman term is small compared to A, and where the Zeeman term is large compared to A.

8. Consider an ensemble of non-interacting ions of spin $\frac{1}{2}$ placed in a magnetic field H. The ions have a magnetic moment μ_o, and are maintained in thermal equilibrium at temperature T.

   a. Calculate the magnetization $M(T,H)$ of the ensemble.

   b. Calculate the entropy S of the system as a function of H and T. Sketch the dependence of S on magnetic field, for fixed T.

   c. From part (a), calculate $(\partial M/\partial T)_H$. From part (b) calculate $(\partial S/\partial H)_T$. Comment on the results, i.e., are these quantities related to each other in a simple way? If so, can you give any general reasons why they are related, or is it an accident that occurs for this special system?

9. Consider a solid with N lattice sites of type A, and N lattice sites of type B. An atom on a type A site has energy $E_A = E$, and an atom on a type B site has energy $E_B = E + e$. The solid is maintained at temperature T. Find the number of atoms that will be found in sites of type B.

PART III - GENERAL PHYSICS QUESTIONS

Ph.D. Qualifying Exam
Fall 1969

Saturday, September 27, 1969
1 - 5 p.m.

There are two sections for Part III. You will have two hours for the first section. Do any ten of the fifteen questions. The questions should be answered briefly but clearly, perhaps with an answer in the form of a short paragraph.

1. Radio signals may be transmitted very long distances over the earth. The reason is that these waves are reflected back to the earth from the ionisphere. Television waves are not reflected from the ionisphere, however. Explain briefly.

2. When one examines the energy distribution of black body radiation in a cavity, the distribution depends only on the temperature of the walls, and is independent of the material from which the walls are constructed. Present a brief argument why this is so.

3. What is the Josephson effect? Describe briefly a use that has been made of this phenomena that has implications far outside the area of solid state and low temperature physics.

4. Why does one side of the moon always face the earth?

5. Hydrogen gas in a laboratory discharge tube shows a discreet spectrum. In a massive system such as a star, it shows nearly a black body radiation spectrum, even though the temperature and pressure in the outer region of the star may be similar to that in the laboratory discharge. Comment on this.

6. Einstein's simple explanation of the photo-electric effect played a major role in establishing the importance of the quantum description of the electromagnetic field. Discuss two features of the data that are easily explained with the quantum theory that are very difficult (if not impossible) to understand with classical physics.

7. What kind of photon energies are required to study crystal structures with x-rays? with electrons? To probe the structure of the proton with electrons?

8. What is the evidence for

   a. The law of conservation of electric charge?

   b. exact equality of proton and electron charge?

   c. stability of the proton?

9. Making use of the equivalence principle (gravitational field and accelerated reference frame have same mechanical results), derive an expression for the gravitational red shift. Consider a radiating source at the top of a tower 500 ft. tall. How big is the "red shift" seen by an observer at the earth's surface? Is it feasible to measure this shift?

10. The "lifetime" of atoms in excited states tends to be roughly 10^{-8} sec. How would you verify the correctness of this statement? Indicate what you consider the major difficulty with such a measurement.

11. Sketch the isotherms of the vander Waal's gas in the P-V plane. If the gas is isothermally compressed, state the rule from which one determines the volume at which liquid begins to form. With great care in cooling, one may sometimes maintain the system in the gaseous state at a volume below the volume just discussed (the supersaturated state). Indicate with an arrow on one of the isotherms the volume below which the supersaturated state cannot be maintained.

12. Which of the following events are allowed by the conservation laws, and which are not? If the event is allowed, what type force (weak, strong, etc.) mediates the interaction? If the event is forbidden, state the relevant conservation law.

a. $\Sigma^+ \rightarrow \pi^+ \Lambda^o$

b. $\Sigma^o \rightarrow \gamma \Lambda^o$

c. $\Lambda^o \rightarrow P\pi^-$

d. $p^o \rightarrow \gamma\gamma$

e. $\pi^- P \rightarrow \Lambda^o K^o$

f. $\pi^- P \rightarrow \Sigma^+ K^-$

g. $Pd \rightarrow PP\pi^o$

13. Present a brief argument that suggests a photon of frequency ν has a momentum equal to $(h\nu/c)$. Give an experimental observation that may be used to confirm that the photon carries mechanical momentum.

14. Discuss two pieces of evidence from which one may conclude that the nucleus does not contain "bound" electrons.

15. Many resonances observed in high energy physics have lifetimes of the order of 10^{-23} seconds. This is about the time required for light to travel the distance of the nucleon radius. Many theories of strong interactions treat resonances on the same footing as particles. How can this approach be justified?

PART III - MATH-PHYSICS

Ph.D. Qualifying Exam
Fall 1969

Saturday, September 27, 1969
1 - 5 p.m.

Do any four of the following problems.

1. Find the Green's function for the operator $\mathcal{L} = \frac{-d^2}{dx^2}$ over the interval (0,1) for functions satisfying the boundary condition $U(0) = U(1) = 0$. If $h''(x) = -f(x)$, find $h(x)$ in terms of an integral over a single variable.

2. Evaluate by contour integration

$$\int_0^\infty \log(1 + x^2) \frac{dx}{x^{1+\alpha}} \qquad (0 < \alpha < 2)$$

3. Calculate by the method of steepest descent the leading term in an asymptotic expansion of

$$H_v^1(2) = \frac{1}{\pi i} \int_{-\infty}^{\infty + c\pi} e^{z \sinh t - vt}\, dt \qquad (|\arg z| < \tfrac{1}{2}\pi)$$

for large z.

4. e[0,1] is defined to be the space of all continuous functions on the interval [0,1], with the inner product of two elements in e defined as

$$\langle b \mid g \rangle = \int_0^1 \bar{b}(t) g(t)\, dt$$

a. Is e, with this innerproduct, a Hilbert space? Justify your answer.

b. Let $\rho(w)$ be a real, positive function, defined on the interval $(-\infty,\infty)$, such that

$$\int_{-\infty}^{\infty} \rho(w)\, dw < \infty$$

r(h) is defined as

$$r(h) = \int_{-\infty}^{\infty} e^{ihw} \rho(w)\, dw$$

Show that r(h) is uniformly continuous on $(-\infty,\infty)$.

c. Show that the operator A defined by

$$(Af)(t) = \int_0^1 r(t-\tau)f(\tau)\,d\tau$$

is Hermitian and positive definite.

5. Define an integral operator K on c by the kernel

$$K(x,\xi) = \xi \quad 0 \le \xi < x$$
$$= x \quad x \le \xi \le 1$$

i.e.

$$(Kf)(x) = \int_0^1 K(x,\xi)f(\xi)\,d\xi$$

Obtain an expansion for $K(x,\xi)$ in the form

$$K(x,\xi) = \sum_{c=1}^{\infty} \psi_i(x)\psi_i(\xi)/\lambda_i$$

and show that the expansion converges uniformly for all $0 \le x,\ \xi \le 1$.

6. We denote by $u(x,t)$ the displacement in the y-direction of a string stretched along the x-axis between the points (0,0) and (1,0). If it is initially at rest on the x-axis, find its displacements under a constant external force proportional to $\sin \pi x$ at each point. Thus, you must solve the boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + C \sin \pi x \qquad 0 \le x \le 1$$

$$u(x,0) = \frac{\partial}{\partial t} u(x,t)\Big/_{t=o} = 0.$$

Physics Qualifying Exam UCI January 15, 1970

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In almost all parts of the exam, you will have choices. Take time to read the problems carefully and decide which ones you want to solve. You will not receive extra credit for solving parts of more than the requested number. If you work on more than the requested number, indicate clearly on your paper which ones you wish to have considered for credit.

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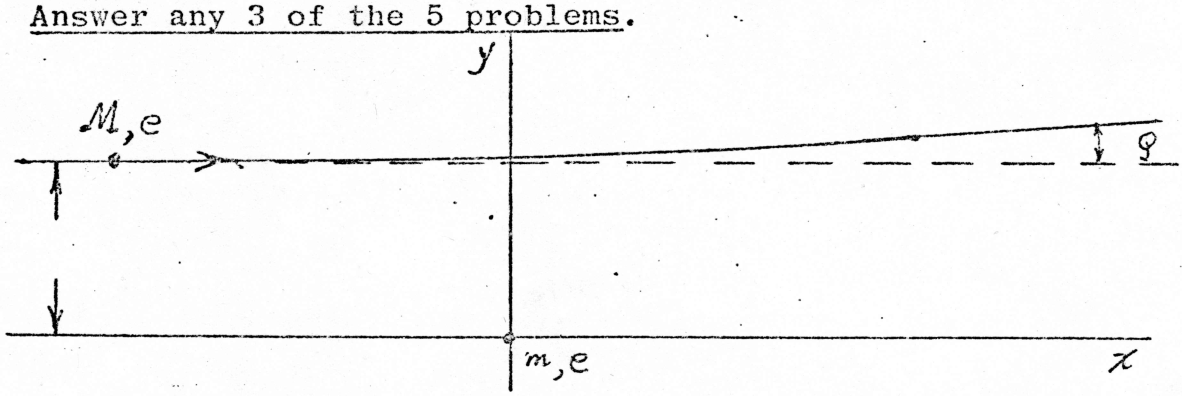
DAY 1

Part I. MECHANICS (1:30)

Answer any 3 of the 5 problems.

1.



A particle of mass m and charge e, initially at rest at the origin, is constrained to move only in the y direction under the influence of a potential $V(y) = \frac{1}{2}Ky^2$. A second particle of mass M and charge e approaches from infinity parallel to the x axis with inital speed v_o and impact parameter s. The incident particle is scattered through a small angle φ ($\varphi << \pi/2$). During the collision process, the motion of the target particle is negligible.

(a) Find φ as a function of v_o and s.

(b) If $\sqrt{\frac{K}{m}}\ \frac{s}{v_o} \ll 1$, find the amplitude of oscillation of the target particle after the collision.

---

2. A system consists of a set of charged particles, all having the same ratio e/m, moving in a uniform magnetic field $\vec{B}$. The interactions considered are the interaction of the separate particles with the magnetic field and a potential energy that depends only on the relative positions of the particles. Let the vector potential describing the field be $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$.

(a) Obtain the Lagrangian for the system.

(b) Express the Lagrangian in terms of coordinate axes rotating about $\vec{B}$ with angular velocity $\vec{\omega}$.

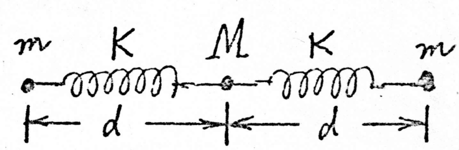
(c) Show that if $\vec{\omega} = -e\vec{B}/2mc$, the Lagrangian reduces to a field-free form if terms of order B^2 are neglected.

(d) What is the significance of this result.

---

3. (The two parts of this problem, although unrelated, comprise a single problem.)

A. In free fall, dv/dt is constant. Galileo says that he first thought that dv/dx is constant in free fall. Discuss thoroughly the kinematics of one-dimensional motion with dv/dx constant. Consider in particular how a body would fall from rest if this were the kinematics of free fall.



B. Two particles of mass m are joined to a co-linear particle of mass M by a pair of identical springs obeying Hooke's law. As shown in the diagram, each spring has force constant K and equilibrium length d. Find the normal mode frequencies for the system.

---

4. A particle incident from infinity scatters from a potential $V = -K \frac{e^{-\alpha r}}{r}$. The particle has energy E and angular momentum $\mathcal{L}$ related by

$$E = V(r_1) + \frac{\mathcal{L}^2}{2mr_1^2} - \Delta ,$$

where Δ is a small positive quantity and r_1 is a root of

$$\frac{\partial V}{\partial r_1} - \frac{\mathcal{L}^2}{mr_1^3} = 0.$$

Discuss the behavior of the orbit for small values of Δ.

Hint: Consider the time needed to move from the turning point of the orbit, r_o, to a larger distance r :

$$\Delta t = \int_{r_o}^{r} \frac{dr}{\sqrt{\frac{2}{m}\left[E - V(r) - \frac{\mathcal{L}^2}{2mr^2}\right]}}$$

5. A particle moves in a circular orbit under the influence of a central force. Examine the motion of the particle if it is slightly perturbed from equilibrium.

(a) Obtain the equations of motion for the deviations of the radial and angular coordinates from their unperturbed values.

(b) Show that if $V(r) \sim r^{-n+1}$, the orbit is stable for $n < 3$.

Part II. ELECTROMAGNETISM (1:30)

Answer any 3 of the 5 problems.

1. Give the integral formulation of the mks equation

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \left(\text{or cgs, } \nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0\right).$$

How would the differential and the integral equations be altered if magnetic monopoles exist? Pay attention to the sign of the magnetic source term and justify your choice of sign, using Lenz's law or any other way of reasoning.

---

2. A system consists of N independent classical damped harmonic oscillators, each having mass m, circular frequency ω_o and damping constant β. If each oscillator has electrical charge q, calculate the frequency-dependent dielectric constant of the system. (Neglect any interaction of one oscillator with another.) Assuming that the dielectric constant differs only slightly from unity, calculate the real and imaginary parts of the refractive index and make a rough plot of each as a function of frequency. Which of the parts is connected with absorption and which with dispersion? This system exhibits "anomalous dispersion." What does this mean?

---

3. Derive an expression for the capacitance of a parallel plate capacitor with plates of area A separated by distance d when the space between the plates is (a) evacuated, and (b) filled with a dielectric material of dielectric constant κ ($\kappa = \epsilon/\epsilon_o$). If the latter capacitor is charged and disconnected from its charging source and the dielectric material is then removed, what happens to (a) the charge on the plates, (b) the $\vec{E}$ field between the plates, (c) the $\vec{D}$ field between the plates, and (d) the potential difference of the plates?

---

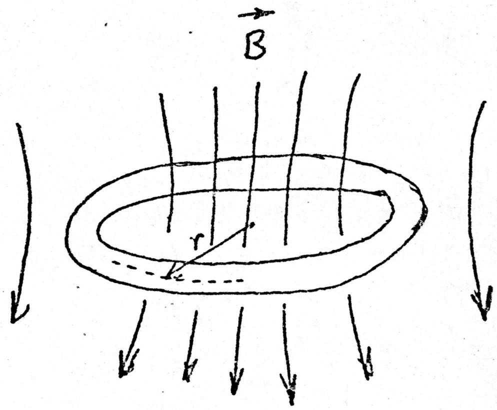
4. A plane electromagnetic wave of circular frequency ω and wave vector $\vec{k}$ is incident normally on the planar surface of a semi-infinite conducting medium of conductivity σ, dielectric constant ϵ and permeability μ. Establish the following:

a) the relation between frequency and wave vector within the conducting medium;

b) the frequency dependence of the attenuation of the electric and magnetic vectors within the medium;

c) for high conductivity, $(4\pi\sigma/\omega\epsilon) >> 1$, the relation between the amplitudes of the electric and magnetic vectors within the medium. Is the energy density mainly electric or mainly magnetic within the medium?

5.



A betatron accelerates relativistic electrons in a doughnut of radius r, using the induction action of an increasing magnetic field $\vec{B}$. All parts of the magnetic field grow in the same proportion; relative strength of different parts do not change with time.

Prove that the betatron will work only if the average field over the area enclosed by the doughnut is twice the field at the orbital radius: $B_{av} = 2\,B_{edge}$.

Physics Qualifying Exam UCI January 16, 1970

DAY 2

Part III. QUANTUM MECHANICS (1:30)

Answer any 3 of the 5 problems.

1. For the one-dimensional motion of a particle of mass m in a potential V(x), develop the WKB approximation, and show how it leads to the following expression for the wave function within the potential:

$$\psi(x) \cong \frac{A}{\sqrt{k}} \exp\left(\pm \int^{x} k\,dx\right), \quad \text{where}$$

$k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$. Using the WKB approximation, obtain a formula for the number of bound states in a square well of dimension L and depth V_o.

2. Consider a free spinless particle of mass m and charge e moving in a constant magnetic field H described by the vector potential (in Cartesian coordinates) A = (-yH, o, o). Write down the Schrödinger equation for the problem. Show that the wave functions can be written as the product of plane waves in the x- and z- directions times a harmonic oscillator wave function involving y. What is the frequency of the harmonic oscillator? What quantum numbers characterize the complete wave functions. What are the energy eigenvalues? Discuss the absorption of electromagnetic radiation by this system.

3. Two protons are bound in a spherically symmetric potential. They occupy two different s- states ($\mathcal{L}$ = o) with radial wave functions $u_1(r)$ and $\mu_2(r)$.

(a) Write an expression for the two-particle wave function (including spin) of their singlet state, taking the exclusion principle into account.

(b) Do the same for their three triplet states.

(c) Treat the electrostatic interaction between the protons as a perturbation. To lowest order, give expressions for the energy shift due to this interaction for both the singlet state and the triplet states.

4. In elementary scattering theory, the wave function is written

$$\psi = A\,[e^{ikz} + f(\theta)e^{ikr}/r]\ .$$

(a) Use the definition of the differential scattering cross-section and the quantum definition of flux to show that

$$d\sigma/d\Omega = |f(\theta)|^2.$$

(b) For s-wave scattering, $f(\theta) = \frac{1}{2ik}\,(e^{2i\delta_o} - 1)$. For a small, perfectly reflecting, sphere of radius $R(kR \ll 1,\ \psi(R) = o)$, find δ_o, and approximate expressions for $d\sigma/d\Omega$ and for $\sigma_T = \int(d\sigma/d\Omega)\,d\Omega$.

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5. Consider a particle moving in the one-dimensional potential

$$V(x) = V_o|x|,$$

where V_o is a constant.

(a) Using appropriate variational trial functions, calculate the energy eigenvalues for the lowest states of even and odd parity.

(b) What matrix element determines the electric-dipole transition rate between these states?

Part IV. THERMODYNAMICS AND STATISTICAL MECHANICS (1:30)

Answer 5 of the 8 problems, including at least 2 labeled T and at least 2 labeled SM.

T 1. Derive the following relations:

(a) $T\left(\frac{\partial S}{\partial T}\right)_V = C_V$, where S is the entropy, C_V the heat capacity at constant volume, T the absolute temperature, and V the volume of a system consisting of a pure substance.

(b) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$, where P is the pressure.

(c) $C_P - C_V = \frac{TV\beta^2}{\kappa}$, where C_P is the heat capacity at constant pressure, β is the coefficient of volume expansion, $\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$, and κ is the isothermal compressibility, $-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$. What conclusions can be drawn from this relation?

T 2.

(a) Derive the Clapeyron equation specifying the temperature derivative of the vapor pressure of a liquid in equilibrium with its vapor.

(b) The vapor pressures of benzene at 20°C and 50°C are 74.7 and 269.0 mm of mercury, respectively. Using appropriate approximations, estimate the heat of vaporization per molecule of benzene. Useful data: $k = 1.38 \times 10^{-16}$ erg/K; density of Hg = 13.6 gm/cm^3.

T 3. Discuss the production of very low temperatures by the adiabatic demagnetization of a paramagnetic salt. Obtain a formula in terms of thermodynamic parameters for the temperature change which takes place during adiabatic demagnetization.

T 4. Define

(a) the Peltier effect,

(b) the Seebeck effect, and

(c) the Thomson effect.

Derive a relation between the Peltier coefficient and the temperature derivative of the Seebeck E.M.F. This relation is an example of Onsager's reciprocal relations. In general, what are Onsager's reciprocal relations?

SM 1. Consider a perfect classical relativistic gas. Using the general equipartition theorem, show that such a gas obeys the usual perfect gas law, $PV = NkT$, where P is the pressure, V the volume, N the number of particles, and T the absolute temperature.

SM 2. A system of N independent electric dipoles, each having a dipole moment μ is placed in a constant external electric field E. Treating the dipoles classically, calculate the mean dipole moment of the system, assuming that the system is maintained at thermal equilibrium at temperature T. By expanding the result in powers of the electric field, derive an expression for the dielectric constant ϵ.

SM 3. Derive an expression for the heat capacity of a solid using Debye's model of an isotropic elastic continuum. Obtain formulas for the heat capacity in the limiting cases of low and high temperatures. How do the results of Debye's Theory compare with typical experimental results, and how may his theory be improved?

SM 4. Discuss Bose-Einstein condensation. Give typical plots of pressure versus specific volume and specific heat versus temperature. What kind of phase transition is involved?

Physics Qualifying Exam UCI January 17, 1970

DAY 3

Part V. MATHEMATICAL PHYSICS (1:30)

Answer any 4 of the 7 problems.

1. (The two parts, although unrelated, comprise a single problem.)

A. Give a sufficient condition on the square matrix B for the validity of $(I - B)^{-1} = \sum_{n=o}^{\infty} B^n$.

B. Consider the equation $\nabla^2\varphi - \lambda\varphi = 0$, $\lambda > 0$, with the condition that $\varphi = 0$ on a closed smooth surface. Show that this equation has no non-trivial solutions inside the surface of proving that

$$\int_V (\nabla^2\varphi)\varphi \, dv < 0$$

(V is the volume enclosed by the surface). How does this condition imply the stated result ?

2. Let $\rho(\omega)$ be real and positive, with $\int_{-\infty}^{\infty} \rho(\omega)\,d\omega < \infty$. Define $r(k) = \int_{-\infty}^{\infty} e^{ik\omega}\rho(\omega)\,d\omega$. Let R be the linear integral operator on $\mathcal{L}^2[0,1]$ defined by

$$|g\rangle = R|f\rangle, \; g(t) = \int_0^1 r(t-\tau)f(\tau)\,d\tau .$$

(a) Show that R is Hermitian and positive definite. (b) Find $r(k)$ if $\rho(\omega) = \dfrac{1}{1+\omega^2}$.

3. For what real values of λ are there continuous bounded solution on $-\infty < x < \infty$ of

$$f(x) = \lambda \int_{-\infty}^{\infty} e^{-|x-y|} f(y)\,dy .$$

Suggestion: Consider what differential equation is satisfied by $f(x)$.

4. Evaluate $\int_0^\infty \log(1+x^2)\,\frac{dx}{x^{1+\alpha}}$.

5. Show that $H_\nu(z) = \frac{1}{i\pi}\int_{-\alpha}^{\infty+i\pi} e^{z\sinh t - \nu t}\,dt$, $|\arg z| < \frac{1}{2}\pi$,

approaches $\sqrt{\frac{2}{\pi z}}\, e^{i(z - \frac{1}{2}\nu\pi - \frac{1}{4}\pi)}$ for large z.

6. Solve $\frac{\partial^2\varphi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$, $0 < x < \infty$, $0 < t < \infty$,

with the condition

$$\varphi(0,t) = f(t) \;;$$
$$\varphi \to 0 \text{ as } x \to \infty;$$
$$\varphi(x,0+) = 0 \;;$$
$$\varphi_t(x,0+) = 0 \;.$$

7. Find a curve y(x) between x = 1 and x = 5 such that

$$\delta\int_1^5 \sqrt{1+y'^2}\,\frac{dx}{x} = 0 \;, \text{ with } y(1) = 0 \text{ and } y(5) = 4.$$

Part VI. RELATIVITY (1:00).

Answer any 2 of the 3 problems.

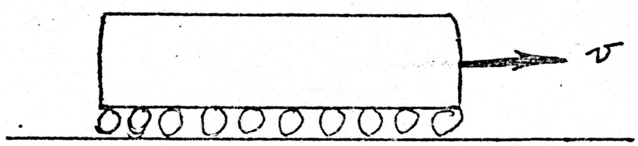
1. The two parts, although unrelated, comprise a single problem.

A. A beam of neutral pions of mass m move through the laboratory with speed $v = \beta c$. Each decays into two photons. What are the maximum and minimum photon energies recorded in the laboratory,

B. By how much does the speed of a 20 GeV electron differ from c? Give the answer both algebraically and numerically. ($m_e c^2 = 0.511$ MeV.)

2. A proton of mass m and total energy E collides with another proton at rest. Derive an expression (in terms of m, E, and c) for the maximum total mass M of the products of the collision. Discuss the two limits of large and small incident kinetic energy. What is the minimum energy E needed for antiproton production (express in terms of $m_p c^2$; a numerical answer is not required)?

3.



A car, whose length when at rest is L_o =6 m, has wheels along its whole length which can be independently braked and can change their speed over the ground instantaneously from a finite value to zero. Two such cars roll at v = 0.80 c.

(a) In Car A, the brakes are applied simultaneously according to observers in the car. How much time elapses during the braking operation according to observers on the ground? In stopping, is the car physically compressed to less than its normal length, or is it stretched to greater length, or does it come to rest with a length of 6 m?

(b) In Car B, the brakes are applied simultaneously according to observers on the ground. According to observers on the train, which are applied first, the front brakes or the rear brakes? How much sooner? Is Car B crushed (compression), torn asunder (tension), or brought to rest with its normal length of 6 m?

Part VII. GENERAL (1:00)

Answer all 12 questions. Short answers suffice, but be precise, not vague.

1. Sketch the nuclear binding energy curve as a function of mass number, and label both axes numerically.

2. Why was it once suggested that energy conservation is violated in beta decay? The nucleus of tritium ($_1H^3$) undergoes beta decay. What are the products of its decay?

3. The electron in a hydrogen atom is replaced by a negative muon. By what factor does each of the following quantities change: the size of the atom? its ionization energy? the wavelengths of its Balmer spectral lines?

4. Explain in terms of a simple experimental set-up why the photoelectric effect makes possible a direct measurement of the ratio h/e.

5. Give the decay modes and the approximate mean lives of the following particles: neutron (n), negative pion (π), positive muon (μ^+).

6. Why does the sun produce neutrinos, not antineutrinos? Are these electron neutrinos or muon neutrinos?

7. What is the DeHaas - Van Alphen effect?

8. A semiconductor has 10^{17} conduction electrons per cm^3. How is the transmission of electromagnetic waves of various frequencies affected by the presence of these free electrons?

9. Baer's law states that rivers in the northern hemisphere erode more on the right bank (looking downstream) than on the left bank, whereas the opposite is true in the southern hemisphere. Explain.

10. Why is it not possible, in principle, to make a 3-dimensional hologram of a moving object, no matter how fast the film?

11. It is possible to propel a boat forward any distance by moving back and forth in the boat. Why does this fact not violate the law of momentum conservation?

12. What is the escape speed from the earth? Give a formula and a numerical estimate. How does it compare with the speed of sound in air?

END

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Day 1 First Morning Session

Part I. MECHANICS (1:30). Answer any 3 of the 4 problems.

1. A two-dimensional linear oscillator consists of a mass (m) coupled through a spring of linear restoring force constant (k) to a fixed origin. Consider the motion limited to a fixed plane containing the origin and therefore possessing two degrees of freedom.

   (a) Determine the natural (angular) frequency (ω) of the system.

   (b) Determine the orbit of the mass (m) and express the general equation for this orbit.

2. One property of an orbit for an inverse-square attractive field is that the total energy (E) [potential plus kinetic] is inversely proportional to the major axis of the ellipse (distance from aphelion to perihelion). Apply this rule to determine the <u>minimum</u> energy (ΔE) required to boost a satellite from an initial circular orbit to a final circular orbit of twice the radius. Assume use of an intermediate "transfer orbit."

   (a) What is the initial energy boost (ΔE_1) required and in what direction should the associated thrust (momentum change) be applied to put the satellite into the transfer orbit? Express in terms of initial energy $|E_1|$ for the original orbit.

   (b) What is the final energy boost (ΔE_2) required to remove the satellite from the transfer orbit and in what direction should the associated thrust be applied?

   (c) What is the total energy change (ΔE) between the initial and final circular orbits?

   (d) Sketch the orbit system.

3. A canonical transformation is one that preserves the Hamiltonian form of the equations of motion. Several tests exist for determining whether a particular transformation is canonical. Apply any such test to establish whether or not the following are canonical:

   (a) $Q = \ln(\frac{1}{q} \sin p)$, $\quad P = q \cot p$

   $P = \ln(\frac{1}{q} \sin p)$, $\quad Q = q \cot p$

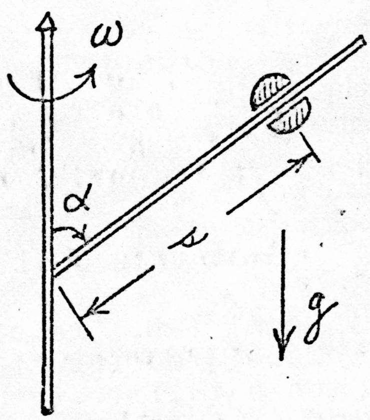
$$Q = \ln(1 + q^{\frac{1}{2}} \cos p), \qquad P = 2(1 + q^{\frac{1}{2}} \cos p)\, q^{\frac{1}{2}} \sin p$$

$$P = \ln(1 + q^{\frac{1}{2}} \cos p), \qquad Q = 2(1 + q^{\frac{1}{2}} \cos p)\, q^{\frac{1}{2}} \sin p$$

(b) For answers of the above that are <u>not</u> canonical, what simple alteration (such as a sign change) would make them canonical?

In each case assume that q and p are canonical coordinate and momentum variables, respectively, in the initial system, while Q and P are the new coordinate and momentum variables.

4. A vertical shaft rotates about its axis at constant angular velocity (ω). A straight rod is attached rigidly to the shaft at an angle (α), as shown, and is carried at the same uniform rotation rate (ω) about the vertical. a bead of mass (m) slides frictionlessly along the rod. Consider gravity (g) directed vertically downward.



(a) Derive the Hamiltonian (H) of the system.

(b) Is this Hamiltonian equal to the energy (E) for the system? Is the Hamiltonian a constant of the motion of the system?

(c) Determine the equation of motion for the system in terms of the distance coordinate (s). At what position (s_o) could the bead balance without motion along the rod?

(d) Solve the equation of motion and specify the conditions under which the motion is i) contained, and ii) unlimited.

Physics Qualifying Exam UCI September 28, 1970

Day 1

Second Morning Session

Part II. ELECTROMAGNETISM (1:30) Answer any 3 of the 4 problems.

1. (a) Write down Maxwell's equations.

   (b) What is the energy density of the fields?

   (c) Assuming that the constitutive equations are linear and using (a) and (b), derive and interpret the expression for the Poynting vector.

   (d) Constant electric and magnetic fields of the form $\underset{\sim}{E} = (E_x, 0, 0)$, $\underset{\sim}{H} = (0, H_y, 0)$ exist in free space. Compute the energy flow across a surface of area 100 m^2 perpendicular to the z-axis.

2. A parallel-plate capacitor with width w, length ℓ, and plate separation d has the region between its plates filled with a dielectric slab of dielectric constant K. The capacitor is charged while it is connected to a potential difference U_o, and then it is disconnected. The dielectric slab is then partially withdrawn in the ℓ-dimension until only the length x remains between the plates.

   (a) What is the potential difference across the capacitor?

   (b) What is the direction and magnitude of the force on the dielectric?

3. A sphere of radius R with a uniform surface charge density σ (rigidly attached) is rotated about an axis through the center with constant angular velocity ω. Show that the magnetic field at an external point is a dipole field and find the equivalent dipole moment.

4. Consider two media separated by the plane $z = 0$. The first medium is a metal whose frequency-dependent dielectric constant is related to the plasma frequency ω_p by

$$\epsilon_M = 1 - \frac{\omega_p^2}{\omega^2} .$$

The second medium is a dielectric whose dielectric constant, ϵ, can be considered frequency independent. A wave propagates along the interface with a scalar potential given

by (Coulomb gauge)

$$\varphi(x,z,t) = 2\varphi_o \cos(kx - \omega t)e^{-k|z|}$$

where k is the wave vector and φ_o is a constant.

(a) Calculate the spatial and temporal dependence of the charge distribution ρ.

(b) Show that the resonant frequency for the wave is given by

$$\omega = \omega_p/\sqrt{1+\epsilon}$$

Physics Qualifying Exam UCI September 28, 1970

DAY 1

Afternoon Session

Part III. QUANTUM MECHANICS (1:30)

Answer any 3 of the 4 problems.

1. An electron of mass μ moves in a one-dimensional potential

$$V(x) = -\frac{\hbar^2 P}{\mu}\,\delta(x^2 - a^2)$$

where P is a positive dimensionless constant, $\delta(x)$ is the Dirac delta-function, and a is a constant length. Discuss the bound states for this potential as a function of P.

2. A particle described by an incoming plane wave

$$\psi_o = e^{i\underset{\sim}{k}_o \cdot \underset{\sim}{r}}$$

scatters elastically from a potential $V(\underset{\sim}{r})$. The outgoing wave can be written as

$$\psi = e^{i\underset{\sim}{k}_o \cdot \underset{\sim}{r}} + g(\underset{\sim}{r}) \xrightarrow[r\to\infty]{} e^{i\underset{\sim}{k}_o \cdot \underset{\sim}{r}} + f(\theta,\varphi)\,\frac{e^{ikr}}{r}$$

where $k = |k_o|$.

(a) Using

$$g(\underset{\sim}{r}) = -\frac{m}{2\pi\hbar^2}\int V(\underset{\sim}{r}')\,\psi(\underset{\sim}{r}')\,\frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}'|}}{|\underset{\sim}{r}-\underset{\sim}{r}'|}\,d\underset{\sim}{r}'$$

derive an expression for $f(\theta,\varphi)$ in the Born approximation.

(b) Find the differential scattering cross section for a particle of charge Ze in a screened Coulomb potential

$$V(r) = Z'e\left(\frac{e^{-\lambda r}}{r}\right)$$

in Born approximation.

3. An atom interacts with electromagnetic radiation with frequency ω in the optical range. For these frequencies the radiation may be treated as a time varying field which is constant in space.

(a) If the E-field has only an x-component

$$E_x = E_{ox} \left(e^{i\omega t} + e^{-i\omega t} \right)$$

find the transition probability from the initial unperturbed state n_o to a state k as a function of the transition electric dipole moment (neglect the B-field).

(b) Show that

$$\Delta J_z = \pm 1, 0$$

for a dipole transition.

4. The magnetic moment of a nucleus (in units of nuclear magnetons) is defined as the average value of the z-component of the magnetic moment taken between states for which $J_z = J$:

$$\mu = \langle \mu_z \rangle_{J_z = J} .$$

For the deuteron (since only the proton contributes to the magnetic moment while both proton and neutron contribute to $\underline{L}$)

$$\underline{\mu} = \tfrac{1}{2} \underline{L} + 2 (\mu_p \underline{s}_p + \mu_N \underline{s}_N)$$

where $\underline{s}_p$ ($\underline{s}_N$) is the spin of the proton (neutron) and $\mu_p = 2.79$, $\mu_N = -1.91$.

(a) Show that the magnetic moment of the deuteron, μ_d, in the states $^3S_1$, $^1P_1$, $^3P_1$, and $^3D_1$ are 0.88, 0.5, 0.69, and 0.31, respectively.

(b) Since the ground state of the deuteron is an eigenstate of parity, it can be an admixture of $^3S_1$ and $^3D_1$. Experimentally, $\mu_d = 0.857$. How much of the ground state wave function is a D wave?

Physics Qualifying Exam UCI September 29, 1970

DAY 2

First Morning Session

Part IV. THERMODYNAMICS AND STATISTICAL MECHANICS (1:30)

A. THERMODYNAMICS. Answer any 2 of the 3 questions.

1. Consider an ideal gas having a ratio of heat capacities at constant pressure and volume of 1.4. One-tenth mole of the gas is initially at a temperature of 100° C in a volume of 1.0 liter. The gas is allowed to expand quasistatically and adiabatically to a volume of 10.0 liters. Calculate:

(a) The initial pressure
(b) The final pressure and final temperature
(c) The work done in the expansion.

2. Wires of two metals A and B are joined to form two junctions, one junction being held at the temperature of melting ice and the other at an arbitrary temperature t° C. The Seebeck E.M.F. has a temperature dependence given by

$$E = (20t + 0.05t^2) \times 10^{-6} \text{ volts.}$$

A current of 0.1 ampere is passed through the junctions. Calculate:

(a) The Peltier coefficient at the junction maintained at $t = 0^\circ$ C

(b) The rate at which Peltier heat is transferred at this junction

(c) The difference in the Thomson coefficients of the two metals near this junction.

3. Ten grams of a paramagnetic salt obeying Curie's law is in a magnetic field of 100,000 oersteds and at a temperature of 10°K. Assume that the specific heat capacity at constant field is constant at 0.10 cal/gm deg. and that the Curie constant is 0.05 deg/gm.

(a) If the field is reduced reversibly and isothermally to zero, calculate the heat transferred.

(b) If the field is reduced reversibly and adiabatically to zero, calculate the temperature change.

B. STATISTICAL MECHANICS. Answer any 2 of the 3 questions.

1. Consider a one-dimensional quantum-mechanical harmonic oscillator with Hamiltonian

$$H = \frac{1}{2m} p^2 + \frac{1}{2} \gamma q^2$$

where p,q are momentum and displacement, respectively, m is the mass and γ is the force constant. Derive an expression for the mean square displacement using a quantum-mechanical canonical ensemble. Make a rough plot of mean square displacement versus temperature. The non-vanishing matrix elements of q are

$$\langle n|q|n+1\rangle = [(n+1)/2\alpha]^{\frac{1}{2}}$$

$$\langle n|q|n-1\rangle = [n/2\alpha]^{\frac{1}{2}}$$

$$\alpha = (m\gamma)^{\frac{1}{2}}/\hbar, \quad n = \text{oscillator quantum number.}$$

2. Consider a semiconductor with two conduction bands. The first band has effective mass $m_1 = 0.05\ m_o$ and energy $E_1 = \hbar^2k^2/2m$, while the second band has effective mass $m_2 = 0.1\ m_o$ and energy $E_2 = \Delta + (\hbar^2k^2/2m_2)$ where $\Delta = 0.4$ ev. The free electron mass $m_0 = 9.11 \times 10^{-28}$ gm and k is the magnitude of the wave-vector.

a) Assuming the electron gas is completely degenerate calculate the electron concentration at which the Fermi energy in band 1 just touches the bottom of band 2.

b) Calculate the total electron concentration at which the Fermi energy is 0.1 ev above the bottom of band 2.

3. A one-dimensional Ising model with cyclic boundary conditions has the Hamiltonian

$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1}, \quad \sigma_{N+1} = \sigma_1$$

where J is the exchange constant for nearest neighbor interactions and the possible values of σ_i are ± 1.

a) Calculate the partition function

b) In the limit $N \rightarrow \infty$, calculate the specific heat at constant volume.

c) Is there a phase transition?

Day 2 Second Morning Session

Part V. MATHEMATICAL PHYSICS (1:30)

Answer any 2 of the first 3 problems and any 2 of the last 3 problems.

1. Show that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(ax+\frac{1}{3}x^3)}\,dx \to \frac{1}{2\sqrt{\pi}}\,a^{-\frac{1}{4}}\,e^{-\frac{2}{3}a^{2/3}}$$

as $a \to \infty$

2. Evaluate the integral

$$\int_0^{\infty} \frac{z^{1/3}}{(1+z)^2}\,dz$$

3. Find the eigenvectors and eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

Are there any degeneracies?

4. Solve the partial differential equation.

$$\frac{\partial u}{\partial t} = \alpha\,\frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions

$u = 0$ for $x = 0$

$u = 0$ for $x = d$

$u = c =$ constant for $t = 0$ and $0 < x < c$

The answer may be left as a series.

5. a. Consider the homogeneous integral equation

$$\lambda f(x) = \int_0^1 xy\, f(y)\, dy \quad , \qquad \begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 1 \end{array} .$$

What are the eigen values $\{\lambda\}$ and the corresponding eigenfunctions of this equation?

b. What is the solvability condition for the inhomogeneous integral equation

$$f(x) = g(x) + \mu \int_0^1 xy\, f(y)\, dy \quad ,$$

where $g(x)$ is a known function and μ is a constant?

c. What does the answer to part b imply about the existence of solutions to the equations

i. $f(x) = x^2 + 3\int_0^1 xy\, f(y)\, dy$

ii. $f(x) = 2x^2 - 1 + 3\int_0^1 xy\, f(y)\, dy$

iii. $f(x) = x + 2\int_0^1 xy\, f(y)\, dy$?

d. What are the solutions to the equations in part c?

6. In the following only square matrices are considered.

a. What is the transpose or adjoint of a matrix?

b. What is a symmetric matrix?

c. What is a Hermitean matrix?

d. What is a unitary matrix?

e. What is an orthogonal matrix?

f. What is the condition that a square matrix have an inverse?

g. What is the condition that the homogeneous matrix equation $\underset{\sim}{M}\,\underset{\sim}{x} = \underset{\sim}{0}$ have non-trivial solutions for the column vector $\underset{\sim}{x}$?

h. What is the condition that the inhomogeneous matrix equation $\underset{\sim}{M}\,\underset{\sim}{x} = \underset{\sim}{a}$ have non-trivial solutions for the column vector $\underset{\sim}{x}$?

i. Show that if M is an n x n Hermitean matrix, it satisfies its own characteristic equation.
[The characteristic equation for a matrix is

$$f(\lambda) = \det(\underset{\sim}{M} - \lambda\underset{\sim}{I}) = 0].$$

j. If $\underset{\sim}{M}_R^{-1}$ and $\underset{\sim}{M}_L^{-1}$ are the right hand and left hand inverses of the matrix $\underset{\sim}{M}$, defined by

$$\underset{\sim}{M}\,\underset{\sim}{M}_R^{-1} = \underset{\sim}{I}$$

$$\underset{\sim}{M}_L^{-1}\underset{\sim}{M} = \underset{\sim}{I} \quad ,$$

prove that $\underset{\sim}{M}_R^{-1} = \underset{\sim}{M}_L^{-1}$.

Physics Qualifying Exam   UCI   September 29, 1970

Day 2   Afternoon Session

Part VI. RELATIVITY (1:00) Answer any 2 of the 3 problems.

1. Two particles of masses m_1 and m_2 scatter elastically. If P and P' are the 4-momenta of m_1 before and after the scattering, respectively, and q and q' are the 4-momenta of m_2 before and after the scattering, respectively, then we can define three invariants $(x^2 \equiv x_o^2 - \underset{\sim}{x}^2)$

$$s = (P+q)^2 = (P'+q')^2$$

$$t = (P-P')^2 = (q-q')^2$$

$$u = (P-q')^2 = (P'-q)^2.$$

(a) Show that $s+t+u = 2(m_1^2 + m_2^2)$.

(b) Express s in terms of the center of mass momentum and t in terms of the center of mass momentum and center of mass scattering angle.

(c) Find an expression for the kinetic energy of m_1 in the laboratory system of m_2 in terms of the total center of mass energy.

2. A relativistic train, traveling with velocity v with respect to ground based observers, has a rest length L. The ground based observers determine its length by noting the position of its front at a particular time and its rear at the same time. According to observers in the car, how much time elapsed between these two measurements that the ground based observers call simultaneous?

3. A thermonuclear explosion on the surface of a planet is observed from a spaceship traveling past at a velocity v nearly that of light. A photograph of the expanding fireball is taken from the spaceship and analyzed aboard the craft. Assume that the fireball expands effectively at the speed of light c in "planet coordinates".

(a) Will the fireball appear spherical (round) in the photograph?

(b) Will the center of the expanding fireball in the photograph coincide with the point of explosion on the planet?

(c) Give the equation describing the shape of the fireball surface as portrayed by (i.e., plotted directly from) the photograph. Compare this with the equation for the fireball in "planet coordinates".

Day 2 Afternoon Session

Part VII. GENERAL (1:00)

Answer all 12 questions. Short answers suffice, but be precise, not vague.

1. Under what circumstances will a particle of mass μ and velocity v ($v<<c$) show distinctly wave properties when it is scattered by a periodic structure of linear period d?

2. Give an order of magnitude estimate of the cross sections for the following reactions:

   (a) $\pi^- p \rightarrow \pi^- p$

   (b) $\gamma p \rightarrow \pi^o p$

   (c) $\nu + n \rightarrow p + e^-$.

3. What is a quark?

4. The differential cross section, $d\sigma/d\Omega$, is known for a process in which the final state consists of two identical particles. Explain how you would find the total cross section.

5. What is the Raman effect? To what do the terms Stokes and anti-Stokes refer?

6. If a cup of tea is stirred at the top, one observes that the tea leaves collect at the bottom in the middle of the cup. Explain.

7. What is Gibbs' paradox and how is it resolved?

8. Explain how low frequency radio waves called "whistlers" can propagate through the ionosphere, whereas ordinary radio waves do not propagate.

9. What is second-sound?

10. Superfluidity occurs in ordinary liquid helium (atomic weight 4). Does superfluidity also occur in liquid of the light helium isotope (atomic weight 3). Why or why not?

11. Whenever electronic mass or electronic charge is determined by measurements involving superconductivity, the result is always twice that for measurements by other means. What is the reason for this?

Part VII. (Continued)

12. A cat suspended upside-down by its paws and released from rest falls under the action of gravity and lands on its feet on the floor. Thus the cat achieves a 180° rotation about a horizontal axis (i.e., the cat's original head-tail direction) despite the complete absence of any external applied torque. Explain how this can occur without violating conservation of angular momentum.

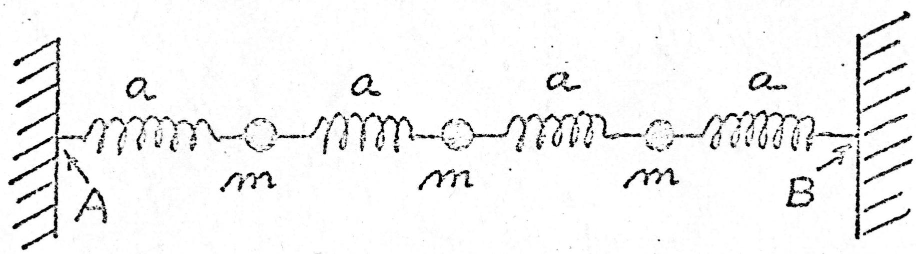
Physics Qualifying Exam UCI May 17, 1971

Day 1 First Morning Session

Part I MECHANICS (2 hours)

Answer any 3 problems

1. Three equal masses m are connected by four identical massless springs. The springs are just without tension in the equilibrium position. The ends A and B



are fixed. The force constant of each spring is k.

(a) Introduce a set of generalized coordinates and find the Lagrangian of the system.

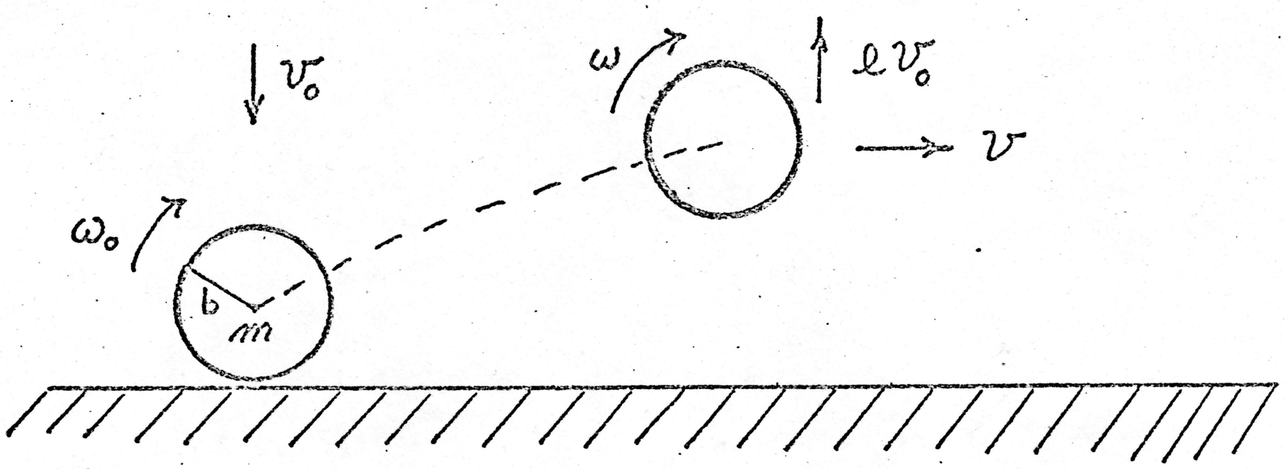
(b) Find the equations of motion.

(c) Find the normal modes for motion along the line joining the masses. Sketch the modes.

(d) Discuss qualitatively what happens to the normal modes if we remove the constraints at A and B.

2. A cylindrical can, open at one end, has height 2a and diameter 2a, and rests with its closed end on top of a rough fixed sphere of radius r. Determine whether or not the equilibrium is stable.

3. A homogeneous sphere of mass m, spinning with an angular velocity ω_o about a horizontal axis, strikes vertically with a velocity v_o on a rough table, the coefficient of friction between the sphere and the table being μ. It rebounds



with an angular velocity ω and the horizontal velocity v. The radius of the sphere is b and the coefficient of restitution is e. What is the largest value of ω_o the sphere may have and still rebound with an angular velocity ω satisfying $b\omega = v$? Express your answer in terms of b, ω_o, v_o, μ, and e.

4. A particle of mass m describes the orbit $r = a(1 + \cos\theta)$ under the action of a force which is always directed toward the origin.

   (a) Find the law of force.

   (b) Determine the total energy of the particle in the orbit.

Day 1 Second Morning Session

Part II. ELECTROMAGNETISM (2 hours)
Answer any 3 Problems.

1. Consider a conducting sphere of radius a in a uniform applied electric field $\vec{E}_o$.

 a) Calculate the electric dipole moment of the sphere.

 b) Suppose many of these spheres are dispersed randomly throughout a dielectric with dielectric constant ϵ. The density of spheres is N/cm^3. Calculate the effective dielectric constant of this system.

2. Consider an electrically neutral, conducting, non-permeable fluid in a uniform magnetic field. Neglect dissipative losses and gravity.

 a) Write the continuity equation, the equation of motion of the fluid and the equation of motion for the magnetic field, in terms of the density ρ, the velocity $\vec{v}$ and the field $\vec{B}$. Define all other symbols carefully.

 b) By assuming small amplitudes, find the phase velocity of the plane waves that can be excited in this system. Consider only the waves propagating parallel and perpendicular to the magnetic field. Which mode is generally called the Alfvén wave? What are the magnetic fields of these waves?

3. Consider an infinitely long uniformly distributed line charge λ per unit length at rest in an inertial frame F' which is moving at velocity $\vec{v}$ with respect to the rest frame F of an observer. The direction of $\vec{v}$ is parallel to the direction of the line charge.

 a) What is the electric field due to this charge distribution at distance d from the line charge, in the frame F'?

b) By transforming the field calculated in a), calculate the magnetic field at the same distance from the line charge observed in frame F.

c) How is this magnetic field related to the magnetic field calculated in F by considering a current λv at a distance d? Why?

4. A point charge Q is located at distance D from a grounded conducting plane. Obtain the induced charge on any area of the plane as a function of the solid angle subtended by the area at the point charge.

Day 1 Afternoon Session

Part III. QUANTUM MECHANICS (2 hours)

Answer any 3 problems

1. A particle moves in a potential V. Suppose the system has eigenstates $|n\rangle$ corresponding to energy eigenvalues E_n. Let x and p_x be the operators representing the canonical position and momentum variables.

   (a) If H is the Hamiltonian, evaluate

   $$[H,x] \text{ and } [H, p_x]$$

   (b) If $|m\rangle$ is a bound state, and $x_{mn} \equiv \langle m|x|n\rangle$, find the value of the sum

   $$\sum_n (E_n - E_m) |x_{mn}|^2 .$$

   (Thomas-Reiche-Kuhn sum rule)

   (c) In what physical processes do the quantities $|x_{mn}|^2$ play a central role?

2. Consider a single electron interacting with two protons which are fixed at a distance R from each other.

   (a) Write the Hamiltonian for the system. (You may ignore any effects due to the spins.)

   (b) Assuming hydrogen-atom wave functions are known, use a suitable approximation procedure to find expressions for the ground-state energy and wave-function of the system. (Integrals need not be explicitly evaluated, but define all quantities carefully.)

   (c) Find analogous expressions for the first excited-state energy and wave-function.

   (d) Estimate the behavior of the ground- and first excited-state energies as a function of R for very large and very small R. Make a rough sketch of these energies as a function of R over the whole range.

3. (a) Suppose it is known that a particle moving in a particular central potential has at least one bound state with orbital angular momentum ℓ. What is the minimum number of bound states the system can have?

(b) If in part (a) it is known that the state with orbital angular momentum ℓ is the highest energy bound state, can you state definitely how many bound states exist?

(c) Consider two spin $\frac{1}{2}$ particles, A and B, which are produced in a singlet spin state and which move apart so that they are no longer interacting with each other. If a measurement of the x-component of the spin of A yields the value + $\frac{1}{2}$, what do you expect for the result of a subsequent measurement of the x-, y-, or z- component of the spin of B, respectively.

(d) Given the premise of part (c), wherein a measurement of $S_x^{(A)}$ yields + $\frac{1}{2}$, what do you expect for the results of a series of measurements of the x-, y-, and z-components of the spin of B, in that order?

4. Plane polarized, long-wavelength radiation is scattered from a free electron.

(a) What is the interaction term responsible for the scattering?

(b) Indicate clearly the steps required for a calculation of the differential cross-section.

(c) Obtain a form for the differential cross-section showing clearly the dependence on the initial and final planes of polarization.

Day 2

First Morning Session

Part IV. THERMODYNAMICS AND STATISTICAL MECHANICS. Answer any two of problems 1, 2, 3 and any two of problems 4, 5, 6.

1. The Hamiltonian of an N particle system is given by

$$H = \frac{1}{2m} \sum_{i=1}^{N} P_i^2$$

where m is the mass of the particle and P_i is the momentum of the ith particle.

(a) Using a classical approach, calculate the entropy of this system confined in a volume V.

(b) Describe the Gibbs paradox using the result obtained in (a) and resolve the paradox.

Note: The volume of an n-sphere of radius R is given by

$$V_n = \frac{\pi^{n/2} R^n}{\Gamma\left(\frac{n}{2} + 1\right)} = \frac{\pi^{n/2} R^n}{\left(\frac{n}{2}\right)!}$$

for n even

2. A rubber band is subject to a stretching force F at temperature T. As a simple model of the rubber band assume that it consists of a linked polymer chain of N segments (N>>1) joined end-to-end. Each segment has length ℓ and can be parallel or anti-parallel to the main axis of the rubber band. Neglect all interactions between segments (except for the fact that they are joined end-to-end) and neglect kinetic energy. Find the length L of the rubber band as a function of temperature T and force F.

3. Using the grand canonical partition function, show that the internal energy of an ideal Fermi gas is given by

$$U = \frac{3}{2} PV$$

where P = pressure, V = volume.

4. The equation of state of a rubber band is

$$F = KT \left[\frac{L}{L_o} - \frac{L^2}{L_o^2} \right]$$

where F is the tension, T the absolute temperature, L its length, L_o its length under no tension, and K a constant. The band is stretched reversibly and isothermally to $2L_o$. Find the amount of heat absorbed or released by the rubber band in the process.

5. Calculate the entropy of a monatomic ideal gas using the equations of state. Calculate the change in entropy as this gas is expanded adiabatically from volume V_1 to V_2 into vacuum. Is this process reversible?

6. A certain gas has equations of state given by:

$$U = Vw(T)$$

$$p = \frac{1}{3} w(T)$$

where U = internal energy
p = pressure
V = volume
w(T) = function of T alone.

Considering the entropy S as a function of temperature T and V,

(a) Find $\left(\frac{\partial S}{\partial T}\right)_V$ and $\left(\frac{\partial S}{\partial V}\right)_T$ as functions of w(T).

(b) Using the mathematical identity

$$\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V},$$

derive the functional form of w(T).
What gas has this form of the internal energy?

Day 2 Second Morning Session

Part V. MATHEMATICAL PHYSICS (2 hours)

Answer any 4 problems

1. (a) Consider the inhomogeneous equation

$$(H - \lambda)\, \psi(\vec{r}) = f(\vec{r})$$

where H is a linear operator and λ a constant. Suppose the complete set of solutions of the corresponding homogeneous equation is known. Demonstrate construction of the solution of the inhomogeneous equation.

(b) If $H = -\vec{\nabla}^2$ and $\lambda = k^2$, find the solution which satisfies the outgoing wave boundary condition.

2. Suppose a function $y(x)$ is constrained to be zero at $x = o$ and $x = a$. How can one find the function $y(x)$ which for fixed arc length encloses the maximum area in this interval? Set up the appropriate equations and describe the procedure for finding the solution.

3. The gamma function is defined by

$$\Gamma(z) = \int_o^\infty e^{-t}\, t^{z-1}\, dt\,.$$

(a) Discuss the analytic properties of $\Gamma(z)$ for Re $z>o$.

(b) Derive a recursion relation relating $\Gamma(z)$ and $\Gamma(z+1)$ for Re $z>o$.

(c) Using this relation to extend the definition of $\Gamma(z)$ to Re $z \le o$, where are the poles of $\Gamma(z)$ located?

(d) What is the residue of the function at each pole?

4. Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{e^{ax}}{e^x+1}\, dx$

where $o<a<1$.

5. Consider a Hilbert space of vectors, $|u\rangle$.

(a) Give the definition of a Hermitian operator.

(b) A Hermitian operator H is defined to be positive definite if $\langle u|H|u\rangle \geq 0$ for any $|u\rangle \neq 0$. Under these circumstances, if $\langle u|H|u\rangle = 0$ what can be said about $H|u\rangle$?

(c) If H and K are two positive definite Hermitian operators, show that $\mathrm{Tr}\,\{HK\} \geq 0$.

(d) If equality holds in part (c), find an expression for the product HK.

Day 2 Afternoon Session

PART VI. GENERAL TOPICS (2 hours)

(Answer any 10 questions)

1. Classify solids in terms of their electronic band structure and give five examples for each group. By looking at a substance can you tell to which group it belongs? If yes, how?

2. Describe the spectrum of electromagnetic radiation from x rays to audio-frequencies by identifying different ranges and specifying the corresponding frequencies and energies (in eV.) Name available sources and detectors for each range.

3. Devise an experimental situation or gedanken experiment which illustrates the relativity of simultaneity, i.e. that the determination of whether or not two events are simultaneous depends on the reference frame of the observer. Describe clearly the results obtained by any observers involved.

4. Explain very briefly the basic principles of the amplification of light by stimulated emission of radiation.

5. Describe in a few sentences (or less) the principal features of each of the following phenomena:

   (a) de Haas-van Alphen effect

   (b) Cerenkov radiation

   (c) Kerr effect

   (d) Meissner effect

   (e) Hall effect

6. Two proton beams of 28 GeV laboratory energy are brought into collision in the CERN intersecting storage rings. Find the laboratory energy that would be required for a single beam incident on a stationary target to give the equivalent center-of-mass energy of the colliding beams.

7. Sagredo: E. Fermi suggested that a possible mechanism for the acceleration of cosmic rays to their enormous energies is the reflection of the particles by magnetic fields within the galaxy.

Simplicio: But any physics undergraduate can tell you that magnetic fields give rise to forces only at right angles to the velocity of a charged particle and cannot, therefore, impart any energy.

Sagredo: (Fill in his reply)

8. Members of the Chinese table tennis team are capable of imparting a velocity of 25 m/sec to a ping-pong ball. What is the energy in GeV of ping-pong balls from this accelerator?

9. The 200-inch reflector at Palomar has been used to beam red laser pulses at the moon. What is the minimum diameter of the beam at the moon 250,000 miles distant?

10. Identify the four known types of forces or interactions which occur in nature and give a measure of the relative strength of each. To the best of your knowledge which of these forces are known to obey invariance under

    (a) Space inversion

    (b) Charge conjugation

    (c) Time reversal.

11. Specify five quantities which are believed to be exactly conserved in all known physical processes. For at least three of these identify the invariance principle associated with the corresponding conservation law.

12. State at least one actual example of empirical evidence for each of the following relativistic effects:

    (a) The contraction of moving rods

    (b) The slowing down of moving clocks

    (c) The equivalence of mass and energy

    (d) The gravitational red shift

    (e) The bending of light in a gravitational field.

PHYSICS QUALIFYING EXAM - U.C.I.

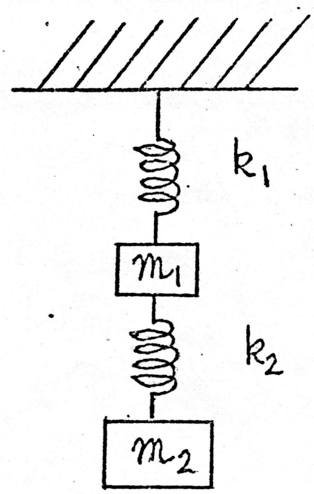
September 27, 1971
Morning Session

Classical Mechanics (2 hours)

Reference Book: Goldstein, Classical Mechanics

Answer any 4 problems

1. Consider the system illustrated below. Mass 1 is held fixed and mass 2 is displaced 2 units of length downward and then both are released from rest. Find the subsequent motion of the system.



The masses move only up and down. Assume $m_1=2$, $m_2=3$, $k_1=10$, and $k_2=20$.

2. Contrary to what you might expect, a satellite orbiting the earth increases its velocity because of the resistance of the earth's atmosphere. Investigate the problem.

3. An early form of electrodynamics used the following velocity dependent potential for a 2-charge system:

$$V = \frac{1}{r} - \frac{(\dot{r})^2}{r}$$

(velocity of light=one)

A) Find the force

B) Find the generalized momentum

C) Write the Lagrangian

D) Find the Hamiltonian

E) Discuss how you would go about determining how a particle moves under such a potential. No solution is required, only a discussion of methods which might lead to a solution.

4. A particle moves under the action of a central force which varies inversely as the cube of the distance from the force center.

A) What conservation laws hold for this motion?

B) Investigate how the particle moves.

5. At one point Dirac was led to consider Lagrangians which were homogeneous linear in the velocities. Investigate the problems associated with deriving a Hamiltonian in such a case.

6. Consider a particle of mass m moving in a central force field described by the potential function $V(\vec{r})$.

a) Show that the following relation must be satisfied if a stable circular orbit is to exist at $r = \rho$.

$$\frac{V''(\rho)}{V'(\rho)} + \frac{3}{\rho} > 0$$

b) Consider the stability of circular orbits in a screened Coulomb potential central field where

$$V(r) = \frac{-k}{r} e^{-r/a}$$

and $k > 0$ and $a > 0$. For what values of the radius of the orbit is the orbit stable?

Physics Qualifying Exam UCI Sept. 27, 1971
Morning Session

ELECTRICITY AND MAGNETISM (2 hours)

Reference book: Jackson, Classical Electrodynamics

Answer any 4 problems.

1. A monochromatic plane electromagnetic wave travels in the +x direction. The electric field is

$$\vec{E} = (0,\ 4 \sin (kx - \omega t),\ E_z)$$

The wave is circularly polarized.

Find E_z. Calculate the $\vec{B}$ field. Calculate the Poynting vector for this wave. State all your assumptions, including choice of units.

2. Maxwell did not use the Lorentz gauge condition commonly used today. Rather, he assumed that the divergence of the vector potential $\vec{A}$ was zero and that the scalar potential was zero. $\vec{A}$ still obeyed a homogeneous wave equation, when no charges or currents were present. Show that these assumptions still lead to Maxwell's equations in free space.

3. An electromagnetic plane wave of frequency ω traveling in a dielectric medium characterized by ϵ_1 and μ_1 is normally incident on a conducting plane characterized by ϵ_2, μ_2 and conductivity σ. Show that in the limit of a good conductor, that is, $\frac{\sigma}{\omega\epsilon_2} \gg 1$, the reflection coefficient is approximately

$$R = \frac{|E_{reflected}|^2}{|E_{incident}|^2} \cong 1 - 2\sqrt{\frac{2\mu_2}{\mu_1}\ \frac{\omega\epsilon_1}{\sigma}}$$

This relation is known as the Hagen-Rubens relation and describes the infrared reflectivity of metals.

4. In an anisotropic medium, we can, by proper choice of axes, obtain linear relations between the components of $\vec{D}$ and $\vec{E}$:

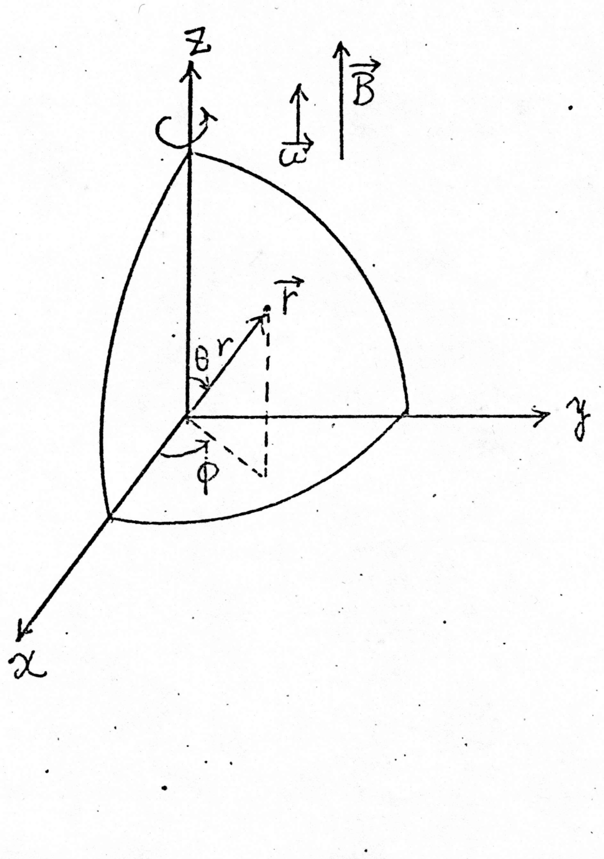
$$D_1 = \epsilon_1 E_1, \quad D_2 = \epsilon_2 E_2, \quad D_3 = \epsilon_3 E_3 \quad .$$

Assume a monochromatic plane wave, with wave vector $\vec{k} = k\vec{n}$ ($\vec{n} = (n_1, n_2, n_3)$ a unit vector in the direction of propagation) and frequency ω. Show that the allowable velocities $v = \omega/k$ must satisfy the Fresnel relation

$$\frac{n_1^2}{v^2 - v_1^2} + \frac{n_2^2}{v^2 - v_2^2} + \frac{n_3^2}{v^2 - v_3^2} = 0 \quad ,$$

where $v_i = c/\sqrt{\epsilon_i}$.

5. A perfectly conducting solid sphere of radius R with zero net charge is set into rotation with a constant angular velocity $\vec{\omega}$ in an external uniform magnetic field $\vec{B}$. $\vec{\omega}$ is parallel to $\vec{B}$.



(a) Show that the electric field at various points r, θ is given by $\vec{E} = E_r\hat{r} + E_\theta\hat{\theta}$ ($\hat{r}$, and $\hat{\theta}$ are unit vectors in direction of increasing r, θ) :

$$\left.\begin{aligned} E_r &= -\omega B r \sin^2\theta \\ E_\theta &= -\omega B r \sin\theta\cos\theta \end{aligned}\right\} r < R$$

$$\left.\begin{aligned} E_r &= \frac{-\omega B R^5}{r^4}\left(1 - \frac{3}{2}\sin\theta\right) \\ E_\theta &= \frac{-\omega B R^5}{r^4}\sin\theta\cos\theta \end{aligned}\right\} r > R$$

(Continued on next page)

5. (Continued)

(Hint: At equilibrium, the total electrical force on the individual charged particle within the sphere is zero.)

(b) Show that there is an induced charge characterized by a distribution within the volume of the sphere of

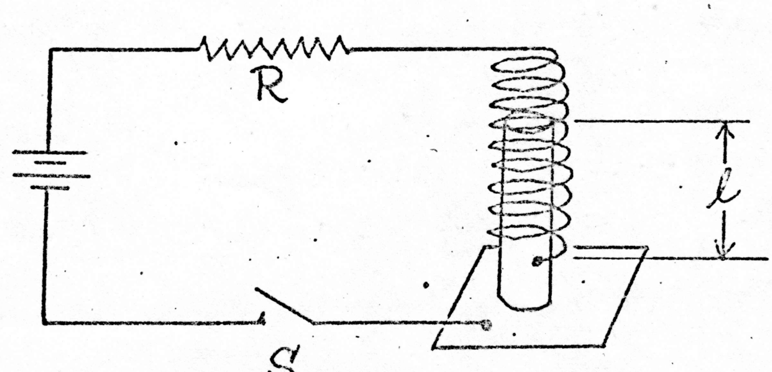
$$\rho = -\ 2\omega B$$

(c) Show that there is a charge distribution on the surface of the sphere of

$$\sigma = \omega BR\ (\tfrac{5}{2}\ \sin^2\theta\ -\ 1)$$

6. Derive the expression for the refractive index for right and left circularly polarized radiation of a medium consisting of N electronic oscillators per unit volume. The oscillators are characterized by elastic force $= -\ k\vec{r}$, where $\vec{r}$ is the radius vector from the center of the orbit to the instantaneous position of the electron. A static magnetic field H_o is oriented along the direction of propagation $\hat{z}$. Neglect damping. What happens if linearly polarized light is sent along the direction of the static magnetic field? This phenomenon is called Faraday rotation. Note that the polarization of this system is $\vec{P} = -\ Ne\vec{r}$.

7. Consider the circuit shown below. The rigid solenoid has length L, N turns, cross-sectional area A. The ferromagnetic rod of negligible resistance has length L, cross section a, permeability $5000\mu_o$, and mass M. The lower end of the solenoid is in electrical contact with this rod and the rod stands on a horizontal conducting plate, thus completing the circuit. Switch S is closed at time t = 0. Find the current in the circuit as a function of time. Consider all possible values of R.



Physics Qualifying Exam | UCI | September 27, 1971 Afternoon Session

Quantum Mechanics (3 hours)

Closed Book.

Answer any 4 problems

1. Derive the quantum mechanical version of the Virial theorem for a particle of mass m moving in a potential $V(\vec{r})$. Compare your result with the corresponding theorem in classical mechanics as to the physical content of the quantities involved.

   Recall that classically the theorem for this case is:

$$\langle T \rangle = -\tfrac{1}{2} \langle \vec{F}\cdot\vec{r} \rangle$$

   where T is the kinetic energy, F the force and $\langle\,\rangle$ denotes time average.

2. Consider a system of N identical atoms each of which has two energy states $|0\rangle$ and $|1\rangle$ separated in energy by $\hbar\Omega$. The life time of the upper state $|1\rangle$ is τ. The matrix elements of the electric dipole operators are given by

$$\langle i|\vec{\mu}|j\rangle = \delta_{ij}\vec{\mu} \;;\; i,j = 0,1$$

   Assume that $\hbar\Omega \gg k_B T$ where k_B is the Boltzmann's constant and T is the temperature of the system.

   i) Assuming that the atoms are all initially in the ground state and that the atoms couple to the EM radiation by a dipole interaction, calculate the rate of energy absorption from a "weak" electromagnetic radiation of frequency $\omega = \Omega$. By "weak" we mean that the relaxation time, τ, times the transition rate is much smaller than unity.

   ii) Now derive a rate equation which gives the number of atoms n in the excited state when the EM radiation is not "weak." At approximately what intensity of the radiation does the energy absorption begin to saturate?

iii) If the volume of this system is V and the atoms are uniformly distributed in this volume, what is the absorption constant of the system for a "weak" EM radiation (i.e., far below saturation)?

3. A hydrogen atom is placed in a crystal at a point where the crystalline electric fields lead to an added potential of the form:

$$V_c(x,y,z) = A\,(x^2 + y^2 - 2z^2)$$

acting on the hydrogenic electron. Here, A is a constant.

a) Working in the representation in which L^2 and L_z are diagonal, find the splitting in the energy levels of the 2p-states as a function of A.

b) Is there any degeneracy for $A \neq 0$? If so, explain the physical origin of the degeneracy. If the potential $V_c' = B(x^2 + 2y^2 - 3z^2)$ is applied in addition to V_c, is any residual degeneracy lifted? Why?

c) What are the eigenfunctions for $A \neq 0$?

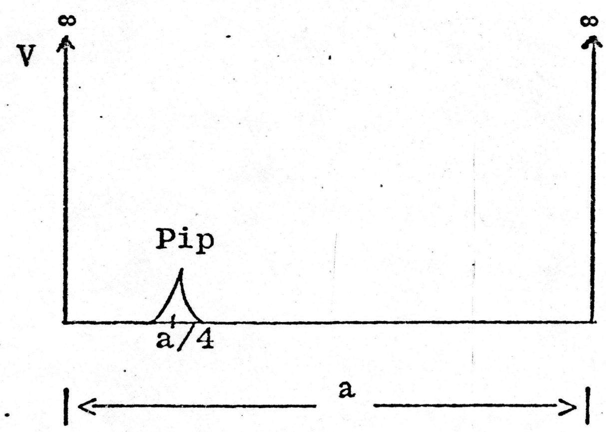
Note: In working out this problem the following functions may be useful:

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

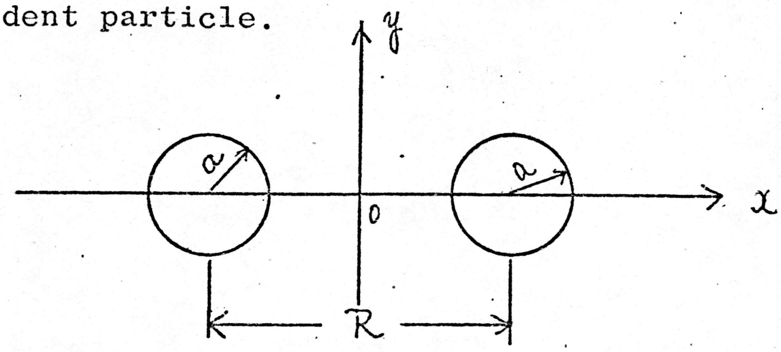
$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\,e^{\pm i\varphi}\sin\theta$$

4. A particle of mass M is in a one dimensional box of length a with infinitely high walls. The box has a very small pip on the floor at point a/4 as shown. The integrated area of the pip is W. What is the probability of finding the particle

in the left hand half of the box in its ground state to the lowest non-vanishing order in the effect of the pip. What expansion parameter must be small to justify this approximation?



5. A particle of mass M is incident on a potential $V(\vec{r})$ that consists of two identical spherical wells. The attractive potential is a constant, V_0, inside the well of radius a. They are separated by a distance R > 2a, as shown in the sketch below. Assume that a is very small compared to the de Broglie wavelength of the incident particle.



A particle propagating initially in the y direction scatters from the potential.

a) Calculate the differential scattering cross section per unit solid angle in the first Born approximation. Note that the differential cross section in the first Born approximation is given by:

$$\frac{d\sigma}{d\Omega} = \left| f_{\vec{k}}(\hat{k}') \right|^2$$

where

$$f_{\vec{k}}(\hat{k}') \approx -\frac{m}{2\pi\hbar^2}\int e^{i\vec{q}\cdot\vec{r}'}\, V(\vec{r}')\,d^3r'$$

$\vec{q} = \vec{k} - \vec{k}'$

$\hbar\vec{k}$ = momentum of the incident particle

$\hbar\vec{k}'$ = momentum of the particle after scattering

$\hat{k}'$ = unit vector in the direction of $\vec{k}'$.

b) Form the ratio of the cross section calculated in part a) to that appropriate for a single isolated well of the same depth and radius. Sketch the behavior of this ratio as a function of the scattering angle for these limiting cases:

i) The de Broglie wavelength of the incident particle is very large compared to R.

ii) The de Broglie wavelength is comparable to R.

iii) The de Broglie wavelength is very small compared to R.

c) State the physical reasons for the behavior you found in part i) and iii) of (b).

6. The Hamiltonian for the Zeeman effect in the presence of spin-orbit coupling is given approximately by

$$H = \beta(\vec{L} + 2\vec{S})\cdot\vec{B} + 2A\,\vec{L}\cdot\vec{S}$$

where β is the Bohr magneton; $\vec{L}$ and $\vec{S}$ are orbital and spin angular momentum operators, respectively; and A is the spin-orbit coupling constant. Note that $[H,L^2] = [H,S^2] = 0$, so that no mixing occurs between states of different (ℓ,s). Now consider one electron in a p-state ($\ell = 1$, $s = \frac{1}{2}$).

a) Describe in words how you would find the energy levels of this electron in the weak field limit and the strong field limit, respectively. What basis functions would you use for each limit? Sketch the energy levels and label them with appropriate quantum numers.

b) Obtain exact energy eigenvalues of this Hamiltonian for ($\ell = 1$, $s = \frac{1}{2}$) by solving secular equations. Work in the representation which diagonalizes L_z and S_z, i.e., $|M_L, M_S\rangle$. [Hint: $[H, J_z] = 0$ where $J_z = L_z + S_z$.]

c) Express the eigenvectors in terms of $|M_L, M_S\rangle$.

The following properties of the operators L_+ and L_- may prove useful:

$$L_+|\ell,m\rangle = \hbar\sqrt{(\ell - m)(\ell + m + 1)}\ |\ell,m + 1\rangle$$

$$L_-|\ell,m\rangle = \hbar\sqrt{(\ell + m)(\ell - m + 1)}\ |\ell,m - 1\rangle$$

where

$$L^2|\ell,m\rangle = \hbar\ell(\ell + 1)|\ell,m\rangle$$

$$L_z|\ell,m\rangle = \hbar m|\ell,m\rangle\ .$$

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Morning Session

Quantum Mechanics (1 hour)

Closed Book.

Answer any 2 problems from (7) through (10).

Answer any 3 problems from (11) through (16).

7. We are given a system described by a Hamiltonian operator H_o. It has N nondegenerate energy levels whose energies are given by $E_1^{(o)}$, $E_2^{(o)}$, ..., $E_N^{(o)}$. (These N levles are the complete spectrum of H_o). We now perturb this system by adding a term V to the Hamiltonian H_o yielding a new Hamiltonian $H = H_o + V$. The energies of the eigenstates of H will be denoted by E_1, E_2, ..., E_N. We are interested in the sum of the energy levels of the perturbed system, which is given by

$$Q = \sum_{S=1}^{N} E_S .$$

One could obviously compute an approximate value Q by calculating each perturbed energy eigenvalue E_S by perturbation theory, starting with the energy $E_S^{(o)}$ of the Sth eigenstate of H_o, and summing the resulting expansion over S from 1 to N.

You are asked to prove that the exact value of Q is obtained if E_s is calculated only to first order in the perturbation V.

8. Using the $1s^2\ 2s^2\ 2p^2$ electronic configuration of Carbon as an example, mathematically justify the Hund's rule which states that the ground state of the configuration has the maximum value of the total spin S consistent with the Pauli principle. You may assume that the exchange integral is positive definite.

9. Consider any L-S level of an atom characterized by a definite J value. Using a group theoretical argument, show that the degeneracy of this L-S level is completely lifted if the atom is placed in a uniform magnetic field.

10. Describe quantitatively two experiments that demonstrate the particle-like behavior of electromagnetic radiation.

ANSWER 3 PROBLEMS FROM THE FOLLOWING:

11. The groups SU_3 and SU_6 have caused much excitement in elementary particle physics over the past few years. How does SU_6 differ from SU_3 (as far as elementary particle physics is concerned).

12. Give the quantum numbers of the initial and final states for the atomic transitions giving rise to the spectral series:

a) Sharp	c) Diffuse
b) Principal	d) Fine .

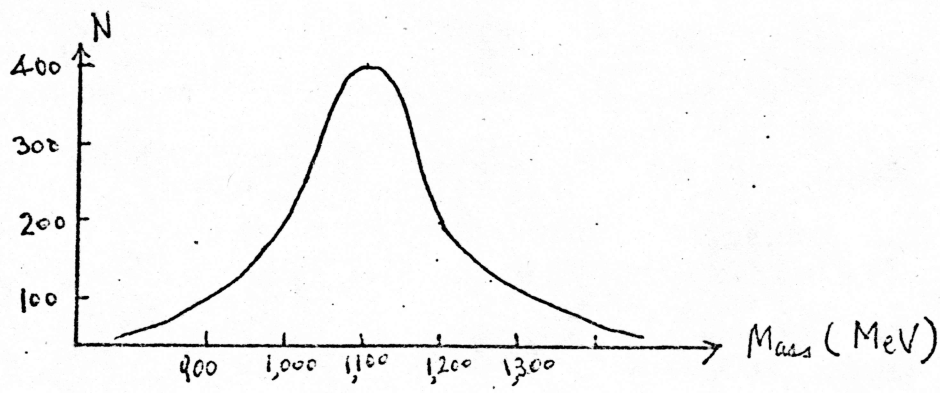
13. Estimate the cross section for the reaction

$$\bar{\nu}\, p \rightarrow n e^{+}$$

where the $\bar{\nu}$ comes from the decay

$$\mu^{+} \rightarrow e^{+}\, \nu\, \bar{\nu} \qquad \text{(at rest).}$$

14. In a particular experiment a very large number of A-mesons is produced. When the masses of all these mesons are histogramed we find:



What is the lifetime of the A-meson (in seconds)? What is the mean free path for decay of an A-meson at 2 GeV/c?

15. It is noted that the linewidth of spectral lines produced by an electrical discharge through a gas becomes broader as the temperature is increased. What mechanisms are responsible for this broadening?

16. The quantum mechanical phase difference between any two points r_1 and r_2 within the bulk of a superconducting material is given by

$$\varphi(\vec{r}_1) - \varphi(\vec{r}_2) = \int_{r_1}^{r_2} \frac{2e\vec{A}\cdot d\vec{\ell}}{\hbar}$$

where $\vec{A}$ is the vector potential and 2e is the charge of the Cooper pair. Show that the magnetic flux contained within a superconducting ring must be an integral multiple of $\frac{h}{2e}$.

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Morning Session

Thermodynamics and Statistical Mechanics (2 hours)

Answer any 4 problems

Reference book: K. Huang, Statistical Mechanics

1. Consider a system of N independent classical one-dimensional anharmonic oscillators with a potential energy

$$\varphi(x) = ax^2 - bx^3 - cx^4 .$$

Obtain an expression for the specific heat which contains contributions of lowest nonvanishing order in b and c. When will this expression be valid?

2. Two boxes containing molecules of spin $\frac{1}{2}$ and magnetic moment μ are joined by a small tube. The respective volumes of the boxes are V_1 and V_2. The molecules are free to move from one box to the other and the total number of molecules is N. There is a constant magnetic field of strength B in box 2. Calculate the ratio of the number of molecules in the two boxes in equilibrium at a temperature T. [Stirling's approximation: $\ln(n!) \approx n\ln(n) - n$]

3. A gas of N particles is contained in a cylinder of cross-sectional area A, but of infinite height. The "gravitational potential" is mgz. Each molecule is a rigid rotator, with energy levels $K(K + 1)\hbar^2/2I$ each $(2K + 1)$ fold degenerate. (I is the moment of inertia.) Each molecule has a nondegenerate electronic ground state, and a 3-fold degenerate excited state at energy ϵ_o (all higher states can be neglected). What is the entropy of this system assuming that kT is large compared to the splitting of translational and rotational states?

4. As is well known, a superconductor can be made to undergo a transition from the superconducting state to the normal state by applying a sufficiently strong magnetic field H. This transition from the superconducting into the normal state is a reversible phase transition in the H-T plane across a threshold curve quite like any other phase transition, for instance, evaporation in the p-T plane. Just as the evaporation phase transition is described by the Gibbs free energy G(p,T), the superconducting transition is described by an analogous thermodynamic free energy g(H,T). It can be shown that

$$g(H,T) = \begin{cases} \Phi_o - \frac{1}{8\pi} H_c^2(T) & |H| \le H_c \\ \Phi_o - \frac{1}{8\pi} H^2 & |H| \ge H_c \end{cases}$$

where H_c is the magnetic field at which the transition occurs and Φ_o is a function only of the temperature.

If $H_c(T) = H_o\left[1 - \left(\frac{T}{T_c}\right)^2\right]$ where H_o and T_c are constants, find (a) the difference between the entropy of the normal and superconducting states as a function of T; (b) the heat released or absorbed per mole at the transition, and (c) the molar specific heat difference between the normal and superconducting states.

5. In a temperature range near absolute temperature T, the tension force F of a stretched plastic rod is related to its length L by the expression

$$F = aT^2 (L - L_o)$$

where a and L_o are positive constants, L_o being the unstretched length of the rod. When $L = L_o$, the heat capacity C_L of the rod (measured at constant length) is given by the relation $C_L = bT$ where b is a constant.

a) Express the change in entropy dS in terms of dE and dL where E is the internal energy of the rod.

b) Compute $\left(\frac{\partial S}{\partial L}\right)_T$ using the appropriate Maxwell relation.

c) Knowing $S(T_o,L_o)$, find $S(T,L)$ at any other temperature T and length L.

d) If one starts at $T = T_i$ and $L = L_i$ and stretches the thermally insulated rod quasi-statically until it attains the length L_f, what is the final temperature T_f? Is T_f larger or smaller than T_i?

6. The equation of state of a Van der Waals gas is

$$(p + \frac{a}{v^2})(v - b) = RT.$$

a) show that the molar energy is

$$E(T,v) = C_v T - \frac{a}{v} + \text{constant.}$$

[Hint: the Maxwell relations may prove useful.]

b) Use this result to calculate the change in temperature in the case of free expansion of one mole of a Van der Waals gas from volume v_1 to volume v_2. (Neglect any temperature dependence of C_v in this calculation.)

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Afternoon Session

Mathematical Physics (2 Hours)

Answer any 4 problems

Reference Book: Any one of the following books-

1) Boas, Mathematical Methods in Physical Sciences

2) Arfken, Mathematical Methods for Physicists

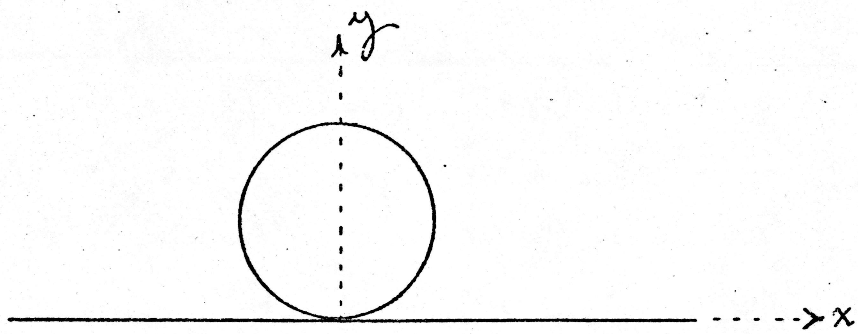
3) Mathews & Walker, Mathematical Methods for Physics

4) Irving & Mullineux, Mathematics in Physics and Engineering

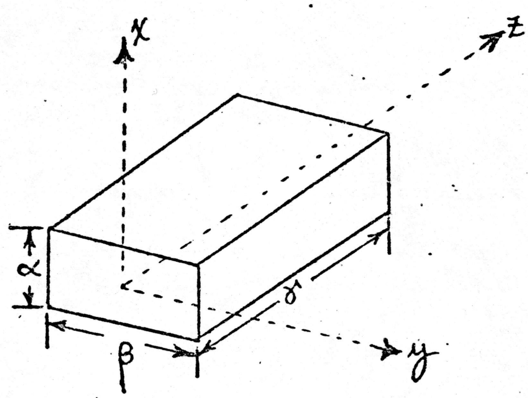
1. Evaluate:

$$\int_{-\infty}^{\infty} \frac{\sin x \, dx}{x^2+4x+5}$$

2. A cylinder (of unit diameter) rests on an infinite plane (with a thin piece of electrical insulation separating them). The cylinder is kept at 100 volts and the plane is kept at 0 volts. Find the electrical potential in the region outside the cylinder. Sketch some of the equipotentials in this region. (Hint: conformal mapping may be useful). - SEE FIGURE ON NEXT PAGE



3. A rectangular bar is initially at constant temperature, T_i. At t=0 it is immersed into a cooling bath ($T=0^o$) and the end at z=0 is uniformly heated at a constant rate (q joules/sec). Find the temperature distribution in the rod T(x,y,z) as a function of time. Assume the thermal conductivity, k, is equal to unity.



4. A unit point mass (m=1) is suspended by a spring (of spring constant k). The mass is in a fluid which exerts a frictional force on the mass which is proportional to the velocity of the mass (the constant of proportionality is c). At t=0 a sharp blow (parallel to the spring's axis) is delivered to the mass (which was originally at rest at the equilibrium position). Assume the impulsive force has the explicit form $F(t)=a\ \delta(t)$. If $c < k/m$, find x(t), the position of the mass as a function of time. Assume that the motion is one-dimensional.

5. Neglecting spin, the electrons of an atom are each specified by the quantum numbers n, ℓ, and m. In free space, the energy of an electron depends only on n and ℓ (assuming each electron moves in a spherically symmetric potential).

If this atom is put in a crystal of octahedral symmetry some of these levels split:

$\ell=0$	(1)	(1)	A_1
$\ell=1$	(3)	(3)	T_1
$\ell=2$	(5)	(3)	T_2
		(2)	E
$\ell=3$	(7)	(3)	T_2
		(3)	T_1
		(1)	A_2

(The numbers in parentheses are the degeneracies of the levels).

In atomic physics you learned the selection rules for electric dipole transitions from one ℓ level to another:

$$\Delta \ell = \pm 1.$$

The problem here is to find the selection rules for electric dipole transitions in an atom when it is in a octahedral crystal (i.e. is $A_1 \leftrightarrow T_1$ allowed?, etc.) The character table for the octahedral group may be found on the last page of this exam.

6. Find the equation of the path of a light ray traveling from (x_1,y_1) to (x_2,y_2) in a medium which has an index of refraction proportional to $1/x$, i.e. $n(x,y) \propto 1/x$. What simple geometric path is this?

7. Let φ be a solution of the differential equation

$$\frac{d^2\varphi}{dx^2} + K^2 \varphi = 0 \qquad 0<x<\ell$$

which satisfies the boundary conditions

$$\varphi(o) = 0 \qquad \text{and} \qquad \varphi(\ell) = \ell\varphi'(\ell).$$

a) What is the equation that the eigenvalues $\{K_n\}$ satisfy?

b) Find the eigenvalues.

c) Find the eigenfunctions $\{\varphi_n(x)\}$ of this differential operator, and normalize them to unity.

d) Find the value of

$$I_{mn} = \int_o^\ell \varphi_m(x)\, \varphi_n(x)\, dx\ .$$

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\,dt$
L27	$\delta(t - a),\quad a \geq 0$ (See Section 7.)	e^{-pa}
L28	$f(t) = \begin{cases} g(t-a), & t > a > 0 \\ 0, & t < a \end{cases}$ $= g(t-a)u(t-a)$	$e^{-pa}G(p)$ [$G(p)$ means $L(g)$.]
L29	$e^{-at}g(t)$	$G(p + a)$
L30	$g(at),\quad a > 0$	$\frac{1}{a}G\left(\frac{p}{a}\right)$
L31	$\frac{g(t)}{t}$ (if integrable)	$\int_p^\infty G(u)\,du$
L32	$t^n g(t)$	$(-1)^n \frac{d^n G(p)}{dp^n}$
L33	$\int_0^t g(\tau)\,d\tau$	$\frac{1}{p}G(p)$
L34	$\int_0^t g(t-\tau)h(\tau)\,d\tau = \int_0^t g(\tau)h(t-\tau)\,d\tau$ (convolution of g and h, often written as $g * h$; see Section 5)	$G(p)H(p)$

L35 Transforms of derivatives of y (see Section 3):

$$L(y') = pY - y_0$$

$$L(y'') = p^2 Y - py_0 - y_0'$$

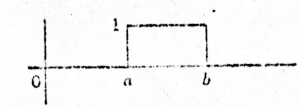
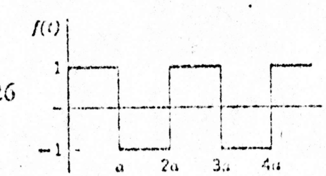
$$L(y''') = p^3 Y - p^2 y_0 - py_0' - y_0'', \text{ etc.}$$

$$L(y^{(n)}) = p^n Y - p^{n-1} y_0 - p^{n-2} y_0' - \cdots - y_0^{(n-1)}$$

The character table for the cubic group is:

O (432)		E	$8C_3$	$3C_2 = 3C_4^2$	$6C_2$	$6C_4$
	A_1	1	1	1	1	1
	A_2	1	1	1	−1	−1
$(x^2 - y^2, 3z^2 - r^2)$	E	2	−1	2	0	0
(R_x, R_y, R_z) (x, y, z)	T_1	3	0	−1	−1	1
(xy, yz, zx)	T_2	3	0	−1	1	−1

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\,dt$	
L1	1	$\dfrac{1}{p}$	$\operatorname{Re} p > 0$
L2	e^{-at}	$\dfrac{1}{p+a}$	$\operatorname{Re}(p+a) > 0$
L3	$\sin at$	$\dfrac{a}{p^2+a^2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L4	$\cos at$	$\dfrac{p}{p^2+a^2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L5	$t^k,\quad k > -1$	$\dfrac{k!}{p^{k+1}}$ or $\dfrac{\Gamma(k+1)}{p^{k+1}}$	$\operatorname{Re} p > 0$
L6	$t^k e^{-at},\quad k > -1$	$\dfrac{k!}{(p+a)^{k+1}}$ or $\dfrac{\Gamma(k+1)}{(p+a)^{k+1}}$	$\operatorname{Re}(p+a) > 0$
L7	$\dfrac{e^{-at}-e^{-bt}}{b-a}$	$\dfrac{1}{(p+a)(p+b)}$	$\operatorname{Re}(p+a) > 0$ and
L8	$\dfrac{ae^{-at}-be^{-bt}}{a-b}$	$\dfrac{p}{(p+a)(p+b)}$	$\operatorname{Re}(p+b) > 0$
L9	$\sinh at$	$\dfrac{a}{p^2-a^2}$	$\operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
L10	$\cosh at$	$\dfrac{p}{p^2-a^2}$	$\operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
L11	$t \sin at$	$\dfrac{2ap}{(p^2+a^2)^2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L12	$t \cos at$	$\dfrac{p^2-a^2}{(p^2+a^2)^2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L13	$e^{-at}\sin bt$	$\dfrac{b}{(p+a)^2+b^2}$	$\operatorname{Re}(p+a) > \lvert\operatorname{Im} b\rvert$
L14	$e^{-at}\cos bt$	$\dfrac{p+a}{(p+a)^2+b^2}$	$\operatorname{Re}(p+a) > \lvert\operatorname{Im} b\rvert$
L15	$1-\cos at$	$\dfrac{a^2}{p(p^2+a^2)}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\,dt$	
L16	$at - \sin at$	$\dfrac{a^3}{p^2(p^2+a^2)}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L17	$\sin at - at\cos at$	$\dfrac{2a^3}{(p^2+a^2)^2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L18	$e^{-at}(1-at)$	$\dfrac{p}{(p+a)^2}$	$\operatorname{Re}(p+a) > 0$
L19	$\dfrac{\sin at}{t}$	$\arctan\dfrac{a}{p}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L20	$\dfrac{1}{t}\sin at\cos bt,\quad a > 0,\ b > 0$	$\dfrac{1}{2}\left(\arctan\dfrac{a+b}{p} + \arctan\dfrac{a-b}{p}\right)$	$\operatorname{Re} p > 0$
L21	$\dfrac{e^{-at}-e^{-bt}}{t}$	$\ln\dfrac{p+b}{p+a}$	$\operatorname{Re}(p+a) > 0$ and $\operatorname{Re}(p+b) > 0$
L22	$1-\operatorname{erf}\left(\dfrac{a}{2\sqrt{t}}\right),\quad a > 0$	$\dfrac{1}{p}e^{-a\sqrt{p}}$	$\operatorname{Re} p > 0$
L23	$J_0(at)$	$(p^2+a^2)^{-1/2}$	$\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
L24	$f(t) = \begin{cases} 1, & t > a > 0 \\ 0, & t < a \end{cases}$ [unit step, often written $f(t) = u(t-a)$]	$\dfrac{1}{p}e^{-pa}$	$\operatorname{Re} p > 0$
L25	$f(t) = u(t-b) - u(t-a)$ 	$\dfrac{e^{-ap}-e^{-bp}}{p}$	All p
L26		$\dfrac{1}{p}\tanh\dfrac{1}{2}ap$	$\operatorname{Re} p > 0$

Physics Qualifying Exam UCI September 28, 1971
Afternoon Session

General Topics (2 hours)

Answer any 10 questions. CLOSED BOOK

Be brief and to the point.

1. List the fundamental interactions known to us at the present time and describe the nature of each. Be specific about the ranges and the strengths. For each of these interactions give an example of a physical phenomenon which is dominated by it.

2. Suppose magnetic monopoles were found to exist. How could electromagnetic theory be adapted to cover this possibility?

3. Discuss the relation between symmetry or invariance principles and conservation laws in classical or quantum mechanics. Discuss more than three such examples.

4. Speculate as to who may win the next Nobel Prize in physics. Support your choice with comments about his contribution to modern physics. If you are unfamiliar with names of individuals, discuss areas of physics.

5. You are given n points in space, with a one ohm resistor connecting each pair of points. Find the resistance between any two points.

6. What are the properties of black holes? (In cosmology, that is!) How would you detect them?

7. What are tachyons? Describe some of their "unusual" properties.

8. Describe the results of the Dickie-Roll-Etvös experiment.

9. Nuclear powered electrical generating stations are constrained to operate at lower temperatures than conventional power stations because of the metallurgy of reactor interiors. For this reason, nuclear power stations generate more thermal pollution than conventional power plans. Justify the latter statement.

10. Describe briefly how the general theory of relativity treats gravity.

11. Why is there essentially no atmosphere on the moon?

12. Why is the harmonic oscillator so important a physical system?

13. Explain, in one sentence, the mechanism in quantum electrodynamics that accounts for the force between charged particles.

14. In what sense is a physical theory "true" or "false"?

Physics Qualifying Exam   UCI   May 15, 1972
9:00 - 11:00 a.m.

MECHANICS

Do 4 of the 6 Problems (2 hours)

1. Consider a simple pendulum consisting of a rigid massless rod of length ℓ hanging from a fixed point. At the end of the rod is a point mass M. Write the Hamiltonian and Hamilton's equations of motion. Solve in the limit of small oscillation.

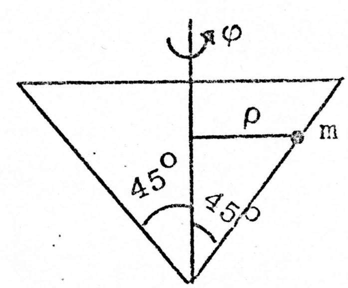
   Now suppose the point of support, which has mass M', oscillates horizontally with frequency ω_o. Write the Hamiltonian.
   Is it constant in time? Why?
   Is it equal to the total energy of the system? Why?

2. A point mass m moves on the inner surface of a right circular cone under the influence of gravity. The cone has a half-angle of 45^o.

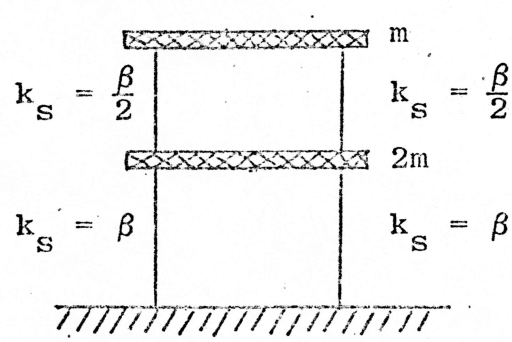
   a) Using general coordinates ρ (radial distance from axis) and θ (angular position), derive the equations of motion.

   b) For what value of ρ will uniform circular motion be possible?

   c) Let ρ equal the value ρ_o for which circular motion is possible. Displace ρ slightly, so that $\rho = \rho_o(1+\Delta)$, where $\Delta \ll 1$. Find the frequency of oscillation of ρ about ρ_o.



3.

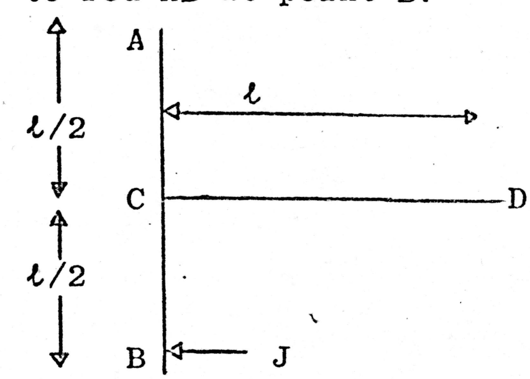


An approximate dynamical model of a 2-story steel-frame building can be constructed from two concentrated masses and two

pairs of massless "shear springs" as shown in the figure. During lateral vibration of the building due to an earthquake, it is assumed that the floors move only parallel to the earth, so that the motion is essentially a shear. If the shear spring constant is k_s, then the potential energy of a spring is $\frac{1}{2}k_s(\Delta\ell)^2$, where $\Delta\ell$ is the net horizontal displacement of the two ends of the shear spring. Find the frequencies and amplitude ratios for the two normal modes.

4. Two identical thin uniform rods, each of mass m and length ℓ, are rigidly joined to form a symmetric T.

The system is initially at rest on a smooth horizontal table. An impulse $\vec{J}$ is applied perpendicular to rod AB at point B. Just after the impulse has struck,



a) What is the velocity of the center of the mass?
b) What is the total kinetic energy of the system, in terms of J?
c) What is the instantaneous axis of rotation of the system?

5. A uniform sphere of mass m, radius b, and $I_{cm} = 2/5\ mb^2$ is initially set spinning with an angular velocity ω_o about a horizontal axis. It is initially in a position infinitesimally above the surface of a horizontal table and it has no translational motion. The spinning sphere is then allowed to come into contact with the table. There is a gravitational acceleration g and a coefficient of friction μ between sphere and table.

a) How much time will elapse before the sphere is rolling without slip?
b) What is the angular velocity after the sphere is rolling without slip?
c) What fraction of its initial kinetic energy was lost while slipping? Where did it go?

6. Consider a rocket to be used for space flight. Let m_o denote the mass of the rocket without fuel and m_f denote the mass of the unburned fuel, giving the rocket a total mass $M \equiv m_o + m_f^o$ initially. Let v_e be the velocity of the exhaust gases relative to the rocket and let $\frac{dm_f}{dt}$ be the (constant) rate of fuel combustion (defined as a positive quantity). Neglect air friction, gravity, and all other external forces.

a) Find the thrust (propulsive force) on the rocket and its maximum speed. Assume we have a chemical rocket, with $v_e \approx 10^4$ m/sec.

b) Now consider photon propulsion, with the advantage of having $v_e \approx c$. We want to give our space travelers the advantage of a large time dilation, so let us choose

$$\gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

equal to 10. Here v is the rocket final speed.

c) Of course, we want our travelers to land safely and return to earth, so actually we will need 3 more such stages for the total of two accelerations and two decelerations. What is the ratio of the mass of the returning rocket remnant to the original total mass of rocket and fuel leaving earth?

Physics Qualifying Exam UCI May 15, 1972
11:15 - 1:15 p.m.

ELECTRICITY AND MAGNETISM

Do 4 Problems

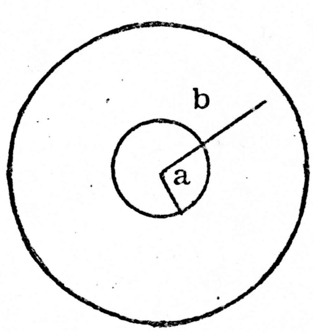
1. A parallel-plate capacitor is charged at a constant rate I (coulombs/sec). The capacitor has circular plates of radius a and a separation h. Neglect edge effects.
   a. Find the total energy (electric plus magnetic) as a function of time. Include only that portion of the magnetic energy which is inside the capacitor volume.
   b. Find the Poynting vector at a distance r from the center of the capacitor
   c. Find the rate of change of energy inside r.
   d. Does the result of b agree with c?
   e. If I is not constant in time, is the answer to d the same? Explain.

2. Find a surface charge distribution on the spherical surface $r = a$ which will produce the potential

$$V = A(x^2 - y^2) + Bx$$

in the region $r < a$. What is the potential in the region $r > a$? [There is no charge anywhere except on the surface $r = a$.] Hint: express V in spherical coordinates, and then in spherical harmonics.

3. Consider a conducting disk of radius b. The conductivity is σ. If the disk has an initial charge density distribution at $t = 0$ of

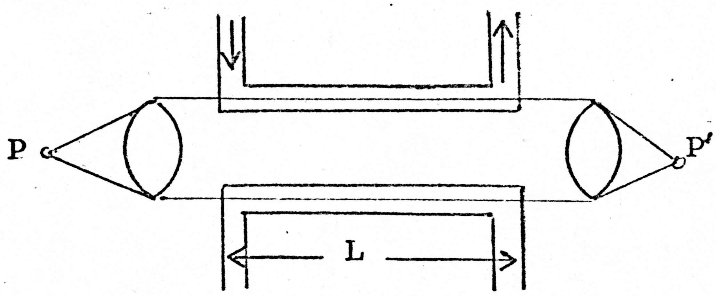


$$\rho = \begin{cases} \rho_0 & r < a \\ 0 & r > a \end{cases}$$

find $\rho(r,t)$ at all other times.

[Hint: Use the continuity equation and $J = \sigma E$.]

4. The core (radius a) of a coaxial cylindrical cable is surrounded by an insulating sheath of conductivity σ_1, outer radius b, and a second layer of conductivity σ_2 extending to the outer metallic conductor of radius c. Find the resistance per meter of cable between the core and the outer conductor.

5. 

Light from a source of frequency f is led through the system shown. If the upper conduit carries a liquid having index of refraction n moving with velocity u, and the lower conduit carries the same liquid at rest, what is the minimum value of u that will cause destructive interference at P'.

6. The ionosphere can be considered as an ionized medium containing N essentially free electrons per unit volume. Show that if a linearly polarized wave propagates in the ionosphere in a direction parallel to that of the small uniform magnetic field H produced by the earth, its plane of polarization will be rotated through an angle proportional to the distance traveled by the wave. Calculate the constant of proportionality. [Hint: calculate the effective index of refraction for right and left circularly polarized light propagating parallel to H. Express the linear polarized light as the sum of right and left circularly polarized light. Express your final answer in terms of $\delta n = n_+ - n_-$ where $n_\pm$ are the two indices of refraction. You need not explicitly evaluate δn.]

Physics Qualifying Exam   UCI   May 15, 1972
2:45 - 5:45 p.m.

QUANTUM MECHANICS (3 hours)

Open Book.

Do any 5 problems.

1. Two identical spin $\frac{1}{2}$ particles of mass m are confined by a one dimensional potential well of length L. ($V(x) = 0$ for $0 < x < L$ and infinite elsewhere). There is a mutual interaction between the particles of the form $V_{12}(x_1,x_2) = \lambda\delta(x_1-x_2)$. To first order in λ, find the energies of the lowest lying singlet and triplet states.

2. A particle with intrinsic spin $S = \frac{1}{2}$ is rotating on a rigid rod. The Hamiltonian of the system is

$$H = \frac{\hbar^2}{2I} L^2 + A\bar{L}\cdot\bar{S}$$

$\bar{L}$, $\bar{S}$ are the orbital and spin angular momentum operators respectively. A and I are constants.

a) What are the energy levels of H?

b) At time $t = 0$ the state of the system is described by the ket vector $|\ell, m_\ell; \frac{1}{2}, m_s\rangle$ with $\ell = 1$, $m_\ell = 1$, and $m_s = -\frac{1}{2}$. What is the probability of finding the system in the same state at a later time t.?

3. The lowest four eigenstates $|i\rangle$, $i = 1,2,3,4$ of a certain Hamiltonian H_o have energies (in unspecified units) of -300, -200, -200 and -100 respectively. A perturbation H_1 is added to H_o. The matrix elements of H_1 between the above four states are (in the same unspecified units).

j \ i	1	2	3	4
1	1	1	3	2
2	1	2	$-\sqrt{6}$	0
3	3	$-\sqrt{6}$	1	4
4	2	0	4	0

The matrix elements of H_1 connecting these four states to all higher states are negligible.

To lowest nonvanishing order find the corrections to the energies of these states.

4. The Hamiltonian of a free particle in one dimension is

$$H = P_{op}^2/2m$$

Let $P_{op}(t)$ and $X_{op}(t)$ be the Heisenberg picture operators for the momentum and position at time t. Evaluate

a) $[P_{op}(t), X_{op}(t')]$
b) $[X_{op}(t), X_{op}(t')]$

5. A particle of mass m and charge e is in the ($n_x = 0$, $n_y = 0$, $n_z = 1$) state of an isotropic harmonic oscillator of natural frequency ω.

a) What is the probability of the particle decaying into the ground state and emitting a photon in the direction θ, φ ?
b) What is the lifetime of this state?

You may assume that $\frac{\hbar\omega}{mc^2} << 1$. Explain why this is a useful assumption for the above problem.

6. The motion of a Dirac particle in a certain potential is governed by the Hamiltonian

$$H = c\,\bar{\alpha}\cdot\bar{p} + \beta mc^2 + \beta U(\bar{r})$$

with $\bar{\alpha} = \begin{pmatrix} 0 & \bar{\sigma} \\ \bar{\sigma} & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(1 is the 2 x 2 unit matrix and the σ's are the usual Pauli matrices). N. B. There is a β in front of U(r).

a) Obtain the nonrelativistic limit of the above Hamiltonian.
b) Do the same for the Hamiltonian

$$H = c\bar{\alpha}\cdot\bar{p} + \beta mc^2 + \gamma_5 U(r)$$

where $\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

7. A proton of mass M is bound in some fixed potential with binding energy E_B. The wave function of this bound state is $\psi(r)$. A fast electron (mass μ) with a large momentum k_i ($\hbar^2 k_i^2/2\mu \gg E_B$) is incident on this system. You may assume that the electron interacts only with the proton.

What is the cross section for knocking out the proton in direction $\hat{k}_p$ and for the electron to emerge with momentum $\vec{k}_f$? Express your answer in terms of the Fourier transform of $\psi(\bar{r})$

$$\varphi(\bar{q}) = \frac{1}{(2\pi)^{3/2}} \int e^{i\bar{q}\cdot\bar{r}} \psi(\bar{r})\, d\bar{r} \quad .$$

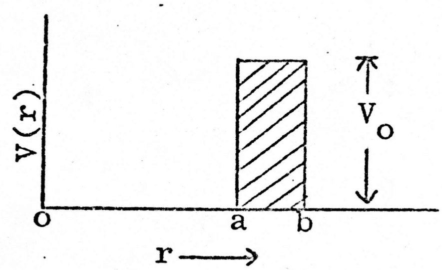
Physics Qualifying Exam UCI May 16, 1972
9:00 - 10:00 a.m.

QUANTUM MECHANICS (1 hour)

Closed Book.

Do 2 Problems.

1. A particle of mass m and angular momentum $\ell = 0$ is trapped in the following potential:



The energy of the particle is $E_o \ll V_o$.

a) Estimate the lifetime of this state.

b) Now assume the particle is incident on this potential from the outside with energy E_o. Estimate the differential cross section.

2. A certain molecule is made out of <u>two</u> atoms of mass M each. The potential acting between the atoms is

$$V(r) = V_o\left(\left(\frac{r_o}{r}\right)^{12} - 2\left(\frac{r_o}{r}\right)^{6}\right).$$

with V_o and r_o given. Discuss quantitatively the low lying spectrum of this molecule.

3. A proton of mass M and kinetic energy E is captured by a heavy spin zero nucleus, Z, leading to the ground state of nucleus (Z + 1) and photon, γ, of energy $E - \Delta$. The spin of the nucleus (Z + 1) is $\frac{1}{2}$. The total cross section for this process is $\sigma(E)$. In terms of $\sigma(E)$ what is the cross section for the reaction

$$\gamma + (Z + 1) \longrightarrow (Z) + p$$

at an incident energy E'. You may treat the motion of the proton nonrelativistically and ignore any recoil of the heavy nuclei.

May 16, 1972

10:15-12:15 AM

Physics Qualifying Exam UCI

THERMODYNAMICS AND STATISTICAL MECHANICS

Open book.

Answer 4 questions.

1. A gas satisfies the van der Waals equation of state

$$(p + a/V^2)\ (V - b) = RT \quad .$$

   a. Find its entropy and internal energy in terms of C_v.

   b. Using an expansion in $1/V$ compute $C_p - C_v$ up to terms of order $1/V^3$.

2. The energy difference between the ^1S_o and ^3S_o states of He is 159,843 cm^{-1}. Taking into account the angular-momentum degeneracies evaluate the fraction of excited (triplet) atoms in Helium gas at 6000^oK.
[Constants: $k = 1.38 \times 10^{-16}$ erg/^oK = 8.6×10^{-5}eV/^oK; 1 eV corresponds to a wave number 8066 cm^{-1}]

3. Show that the chemical potential of a photon gas is zero.

4. An electron in a magnetic field $\vec{H}$ has an energy $\pm\ \mu_B H$, depending on the spin orientation relative to H.

   Calculate the spin paramagnetic susceptibility of a completely degenerate Fermi gas at absolute zero.

5. The canonical partition function for a certain system of N particles at a temperature T and volume V is

$$Q_N(V,T) = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N \left\{1 + 2^N e^{-V/\lambda^3}\right\}$$

$$\lambda = \sqrt{(2\pi\hbar^2)/(mkT)}$$

   Using either the canonical or grandcanonical partition function (grandcanonical much, much easier), find the equation of state of this system. Sketch a P-V isotherm and discuss its salient features.

6. Find the density distribution of a gas in a cylinder of radius R and length ℓ rotating around its axis with angular velocity ω, assuming that the total number of particles is N.

May 16, 1972

1:45 - 3:45 PM

Physics Qualifying Exam UCI

MATHEMATICAL PHYSICS

Closed Book

Do 5 Problems

1. a. Show that the "harmonic oscillator" wave-functions

$$\Psi_n(x) = (-1)^n(2^n n!\pi^{\frac{1}{2}})^{-1/2}\, e^{x^2/2}\, \frac{d^n}{dx^n}\, e^{-x^2}$$

are, up to a constant, their own Fourier transforms.

b. What is the meaning of the relation:

$$\sum_{n=0}^{\infty} \Psi_n(x)\Psi_n(y) = \delta(x-y) \quad ?$$

c. Assume that any function f, such that $\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$,

can be expanded into a series with respect to the Ψ_n.
$f(x) = \sum_{n=0}^{\infty} C_n\Psi_n(x)$, such that

$$\lim_{N\to\infty} \int_{-\infty}^{\infty} |f(x) - \sum_{n=0}^{N} C_n\Psi_n(x)|^2 dx = 0 .$$

Compute the Fourier transform

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-ikx}dx$$

in terms of the $\{C_n\}$ and show that

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\tilde{f}(k)|^2 dk$$

2. Use the method of steepest descent to derive Stirling's formula

$$\Gamma(\lambda+1) = \sqrt{2\pi\lambda}\,(\lambda/e)^{\lambda}\{1+O(1/\lambda)\}$$

with $\Gamma(\lambda+1) = \int_0^{\infty} x^{\lambda}e^{-x}dx$

3. Show that

$$\int_0^{2\pi} \frac{dt}{(a+b\cos t)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}} \qquad (a>b>o)$$

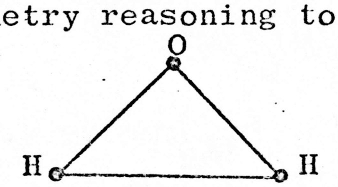
4. Use Fermat's principle to show that light-rays starting from the origin in a (two-dimensional) medium with index of refraction

$$\frac{v}{c} = \frac{1}{\sqrt{A-y}} \qquad (A = \text{constant})$$

are parabolas.

What is the mechanical analog of this problem?

5. Find the normal modes of vibration of an H_2O molecule (see Figure). You may use either group - theoretical methods (the character table of C_{2v} is given below) or qualitative symmetry reasoning to determine the three modes.



C_{2v}:

E	C_2	σ_v	σ_v'
3	1	3	1

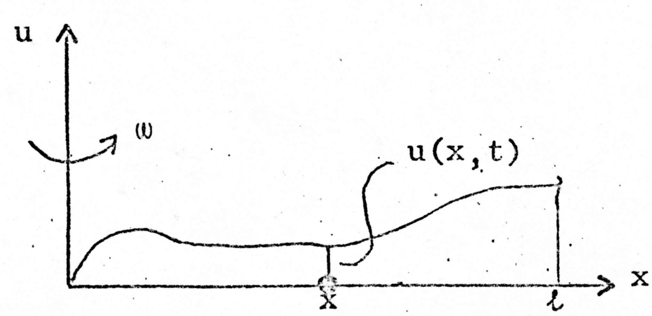
(Here C_2 leaves O in place, σ_v is the reflection leaving all atoms in place, σ_v' leaves only O in place.)

6. Discuss the conformal mapping

$$z = \frac{h}{\pi}\left(e^{\frac{\pi}{V}w} + \frac{\pi}{V}w\right)$$

with h and V constants. What electrostatic interpretation can you attribute to the h, V, Im w ?

7. Study the small vertical vibrations of a homogeneous, perfectly flexible string of length ℓ, fixed at one end, free at the other and rotating around the fixed end in a horizontal plane with fixed angular velocity ω.



(Continued)

7. (Continued)

a. Using the notation in the figure and assuming that the centrifugal force exerted at the point x, which is equal to $m\omega r^2$, with $m = \rho(\ell - x)$, $r = \frac{\ell + x}{2}$, is the only restoring force, show that the equation of motion is

$$\frac{\partial^2 u}{\partial t^2} = \frac{\omega^2}{2} \cdot \frac{\partial}{\partial x} \left[(\ell^2 - x^2) \frac{\partial u}{\partial x} \right] \qquad (1)$$

with boundary conditions

$$u(0,t) = 0 \ , \ u(\ell,t) = \text{finite} \qquad (2)$$

b. Find the normal modes of vibration.

c. Find the motion of the string if the initial conditions are

$$u(x,0) = \alpha x$$

$$\frac{\partial u(x,0)}{\partial t} = 0 \ .$$

Necessary formulas:

$$P_n(\xi) = \frac{1}{2^n n!} \frac{d^n (\xi^2 - 1)}{d\xi^n}$$ are solutions of the

equation

$$\frac{d}{d\xi} \left[(1 - \xi^2) \frac{dy}{d\xi} \right] + n(n+1) y = 0$$

with boundary conditions:

$$y\ (\pm 1) = \text{finite} \ .$$

$$\int_{-1}^{+1} P_n(\xi)\ P_m(\xi)\ d\xi = \frac{2}{2n+1}\ \delta_{nm}$$

$$P_{2n-1}(0) = 0 \quad P_{2n}(0) = (-1)^n \frac{(2n-1)!!}{(2n)!!}$$

8. Use Rayleigh-Schrödinger perturbation theory to determine the change in the fundamental frequency of a rectangular membrane $0 \le x \le a$, $0 \le y \le b$, produced by a small nonuniformity of the mass-density of the form

$$\rho(x,y) = \begin{cases} \rho_o + \epsilon & 0 \le x < \frac{a}{2} \\ \rho_o & \frac{a}{2} < x \le a \end{cases} \ , \ 0 \le y \le b$$

$$\rho_o = \text{constant}$$

8. (Continued).

[Hint: The normal modes satisfy Helmholtz' equation

$$\nabla^2 u + \omega^2 u = 0 \quad \text{with} \quad \omega^2 = \frac{k}{\rho}$$

where k is the tension. Use the expansion

$$\frac{k}{\rho_o + \epsilon} = \frac{\omega^2}{1 + \frac{\epsilon}{\rho_o}} = \omega^2 \left(1 - \frac{\epsilon}{\rho_o}\right)$$

assuming $\frac{\epsilon}{\rho_o} << 1$, before applying perturbation theory].

Physics Qualifying Exam UCI May 16, 1972
4:00 - 6:00 p.m.

GENERAL

Answer any 10 questions

1. We all say that the photon has zero rest mass. Experimentally how well is the rest mass known? Describe an experiment to measure the photon rest mass.

2. What is a parton? A quark?

3. I measure the resistivities of potassium and silicon as functions of temperature near $300^{\circ}K$. Why do the two $\rho(T)$ curves have slopes of opposite sign? For each material what is the functional form of $\rho(T)$ near $0^{\circ}K$?

4. While floating near the moon in your space suit, you encounter an extraterrestial being also in a space suit. You hold out your right hand to shake and he holds out his left. Why do you not shake hands with him? Describe how you might communicate with him, using only unpolarized electromagnetic radiation, and learn if it is safe to shake hands.

5. The concentration of CO_2 in the earth's atmosphere is increasing at a rate of almost one part per million per year. The resulting direct effect on the earth's energy balance is called the greenhouse effect. Describe it.

6. A stick of length L_1, measured in its rest frame, slides along an ice surface with velocity v (very close to c) in the direction of its length. It approaches a hole in the ice a distance L_2 across, in the frame of the ice, where L_2 is much less than L_1. However, the stick is moving so fast that

$$L_2 \gg L_1 \sqrt{1 - \frac{v^2}{c^2}},$$

and consequently the stick falls in. How would an observer on the stick describe this?

7. Describe the transistor and briefly state its principles of operation.

8. You receive an article in which the author proposes to build an EASER (electron amplification by stimulated emission of radiation), that is, an electron analog of the laser. Why do you immediately throw the article away?

9. Relate Cherenkov radiation and the mechanism causing sonic boom in aircraft. Could Cherenkov radiation be seen with gravity waves?

10. In the Fizeau experiment light is propagated through a moving medium, such as water flowing through a pipe. What is the speed of propagation measured in the laboratory and what does this result require of the "ether"?

11. Describe how the Hanbury Brown-Twiss intensity interferometer (R. Hanbury Brown and R. Q. Twiss, 1956) can be used to measure stellar diameters. How does it differ from the Michelson stellar interferometer?

12. Write a sentence or two defining each of 3 of the following:

a) Roton

b) Nematic liquid crystal

c) Strangeness

d) Dilution refrigerator

e) Neutrino

f) Tardyon.

QUANTUM MECHANICS I

1. Consider a 2 state quantum mechanical system whose Hamiltonian is H_1 and whose eigenstates are $\psi, \bar{\psi}$. At time T_o H_1 is turned off and $H_2 = H_{2_o} + \Delta H_2 \neq H_1$ is turned on. The eigenstates of H_{2_o}, ψ_1, ψ_2 (linear combinations of $\psi, \bar{\psi}$) have energies E_1, E_2 and decay with time constants Γ_1, Γ_2 (due to ΔH_2). We observe this system at some time $t > T_o$. Compute the probability for finding ψ and $\bar{\psi}$ at time t, if at $t = T_o$ the particle is in the state ψ.

2. A particle moves in one dimension with energy $E > 0$, in the presence of the potential $V(x) = -\lambda\delta(x)$.

   a. Find an exact expression for the scattering amplitude, and the probability that the particle is reflected from the potential.

   b. From the properties of the scattering amplitude in (a), deduce the energies of any bound states that may be associated with the potential.

3. A particle with spin $\frac{1}{2}$ is placed in a D.C. magnetic field $\hat{z}\, H_o$ along z with its spin up (in the + z direction). At $t = 0$, a constant D.C. magnetic field of strength h is applied parallel to the x axis, and the field h remains on for all times $t > 0$. Find an explicit expression for the wave function of the spin for times $t > 0$.

4. A particle of mass M moves in the harmonic oscillator potential $\frac{1}{2} kr^2$ in three dimensions. An electric field of strength E_o is applied parallel to the z axis, and the particle has charge e.

a. In the presence of the field, find the form of the eigenfunctions and eigenvalues.

b. Suppose the particle is in the ground state. By comparing $\langle \vec{r} \rangle$ in the presence and absence of the field, find the magnitude of the electric dipole moment induced by the field.

5. A particle is governed by the harmonic oscillator Hamiltonian

$$H = \frac{1}{2} P^2 + \frac{1}{2} \Omega_o^2 x^2$$

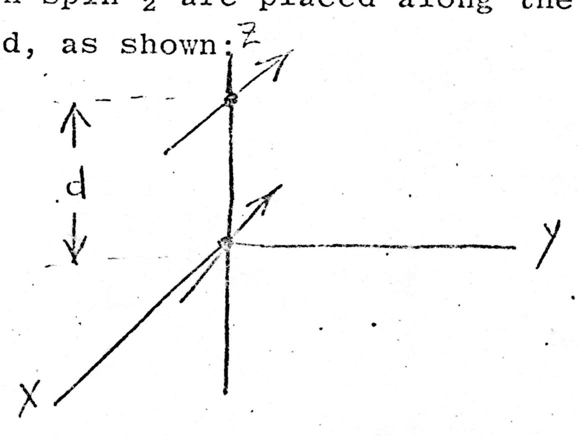
a. If x(t) is the position operator in the Heisenberg representation, find an explicit form for x(t) in terms of operators in the Schrödinger representation.

b. At time t = 0, the particle is placed in the state

$$\varphi(x) = \frac{1}{\sqrt{\pi^{\frac{1}{2}} \Delta}} \exp[-(x-x_o)^2/2\Delta^2] .$$

If $\langle x \rangle$ is the expectation value of the position operator, find an expression for $\langle x \rangle$ as a function of time. (Note that the wave function $\varphi(x)$ is properly normalized in the statement above.)

6. Two fermions with spin $\frac{1}{2}$ are placed along the z axis separated by the distance d, as shown:



They are described by the Hamiltonian

$$H = -H_o(\sigma_1^z + \sigma_2^z) + J\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + K(\vec{\sigma}_1\cdot\vec{\sigma}_2)^2$$

where J is positive and considerably larger than either H_o or K.

a. Find the energy levels and eigenfunctions of H and draw an energy level diagram, indicating which eigenfunction is associated with each energy level.

b. The fermions are in the ground state, in the energy level scheme of (a). A magnetic field of frequency Ω and wave vector q propagates parallel to the z axis:

$$\mathcal{H}(\vec{x},t) = h_o(\hat{x} + i\hat{y})e^{iqz}\,e^{-i\Omega t} \quad .$$

Indicate which transition in the diagram of part (a) can be induced by a field of this form, and calculate the matrix element required for a computation of the transition probability.

c. If the two fermions are the protons in an H_2 molecule, and if the field is a microwave field, would you expect the transition considered in (b) to be strong or weak, compared to the NMR signal of an isolated proton? You should not have to do any calculation to answer this, and one sentence will suffice.

7. An electron is placed initially into an eigenstate $\varphi_o(\vec{x})$ of the Hamiltonian H_o. The electron is subjected to a weak electric field $E_o(t)$ applied parallel to the x axis with the time dependence

$$E_o(t) = \begin{cases} E_o & -\frac{T}{2} < t < +\frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \quad .$$

a. Find an expression for the probability that the electron is found in the state $\varphi_n(\vec{x})$ after the time + T/2. You may assume the field E_o is weak, and that all eigenstates of H_o are non-degenerate.

b. Imagine this result applied to a hydrogen atom. What is required of E_o and T for perturbation theory to be valid? You should be able to answer this from the structure of the answer to (a), without detailed calculation.

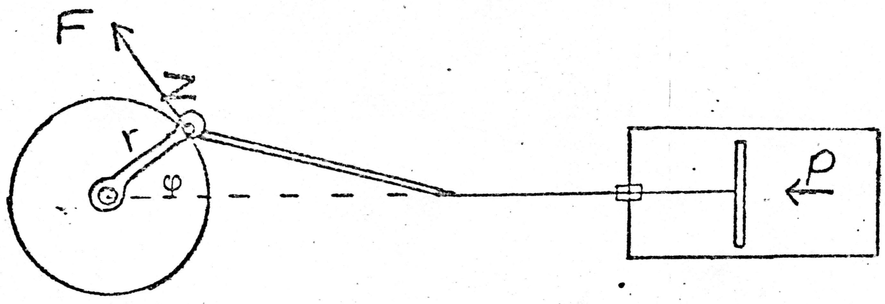
PHYSICS QUALIFYING EXAM UCI October 4, 1972

CLASSICAL MECHANICS

1. Consider a river which flows from North to South. Compute the angle which the normal to the water surface makes with a plumb bob.

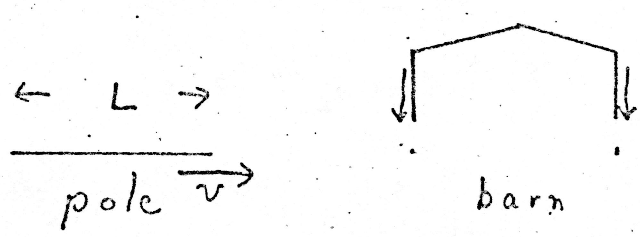
2. Consider the scattering of point particles of mass m from a perfectly hard sphere of radius R and mass M. Compute the differential and total cross section in the center of mass frame. Compute the total cross section in the laboratory frame.

3. Consider the drive mechanism of the steam engine illustrated in the sketch below. If P is the steam pressure in the cylinder, compute the force F developed at the crank pin Z.



4. Do one of the two parts only. Explain carefully and quantitatively.

a. Observer A sits in the rest frame of a barn L meters long with doors at either end, which are open. A pole vaulter B carries a pole of length L at velocity v, and runs into the barn.
Observer A has the barn doors closed when both B and the pole are in the barn, thus containing B.



4. (cont'd.)

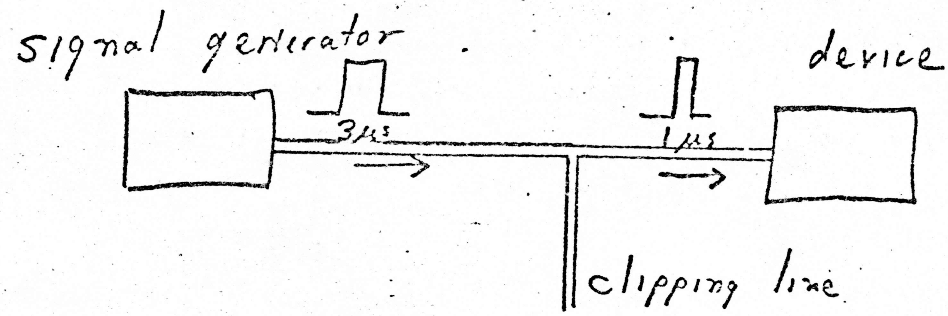
On the other hand, B sees the length of the barn to be less than L, since it is foreshortened by a factor γ. He knows that if the barn doors are closed simultaneously, one door or the other would strike the pole. What is the resolution of this apparent difference of opinion?

b. Explain the twin paradox. Twin A remains on earth, and twin B travels 1.5×10^{12} meters at the velocity c/2, then stops and returns to earth at the same speed. Which twin is older, and by how much? Why is this problem called a paradox, and why is it not? Discuss the aging of each twin at each stage of the trip.

5. A particle moves in a circular orbit under a central force F(r) directed from the center of the circle. Obtain an equation of motion for small radial displacements of the particle from the circular orbit. Obtain a condition on F(r) and dF(r)/dr for stable oscillations about circular orbits. Determine which power laws permit stable orbits.

6. a. Write down the wave equation for a string of linear mass density ρ and tension T. Consider an infinitely long continuous string which, for $X < 0$ has linear mass density ρ, but has a density $\rho_2 > \rho_1$, for $X > 0$. A wave train oscillating with frequency ω is incident from the left. Find the reflected and transmitted intensities and phases.

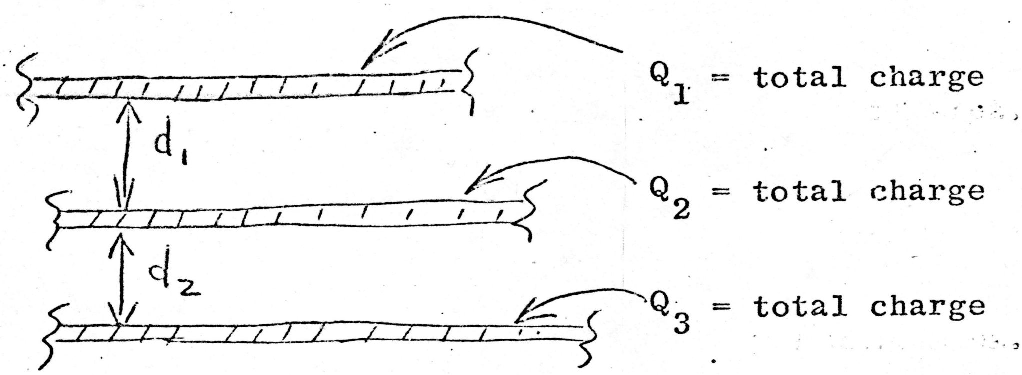
b. Solve the following experimental problem. A physicist wishes to pulse a piece of equipment which is capable of responding to a one μsec. long positive square pulse. However he has a signal generator which is capable only of producing a 3 μsec. long square pulse. He attempts to use a "clipping line".



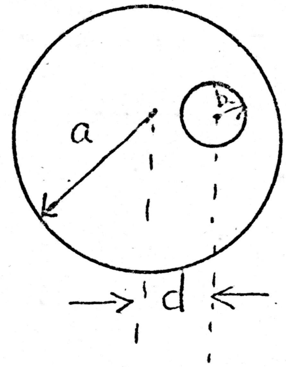
How should he choose the parameters of this setup to obtain the desired result. (He uses transmission lines of propagation velocity c/2.)

ELECTRICITY AND MAGNETISM SECTIONS

1. Three metal plates, each of large area A, carry total charge Q_1, Q_2, and Q_3, as indicated in the sketch below. Find the total charge on each <u>side</u> of the three plates.



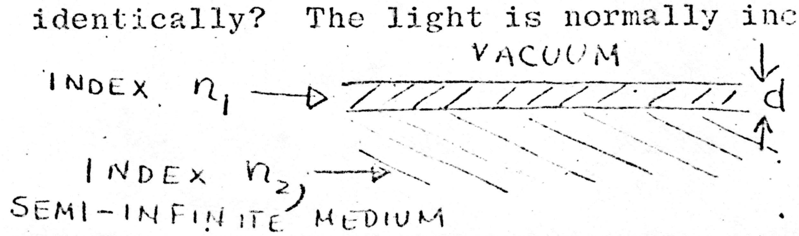
2. Consider a wire in the following shape:



The wire has a circular cross section of radius a, with an off-center hole of radius b < a as shown.

The wire carries a current in the form of a uniform current density J. Find an expression for the magnetic field in the hole. Draw a sketch of the lines of H in the hole. (Hint: Think of the superposition principle.)

3. Light of frequency ω falls on the layered dielectric structure illustrated in the sketch below. How thick should the layer of index of refraction n_1 be for the reflectivity to vanish identically? The light is normally incident on the structure.



4. A particle of charge Q is placed at rest at the origin of the coordinate system at the time $t = 0$. A magnetic field H_o is present parallel to the z axis, and an electric field of strength E_o is parallel to the x axis. Solve for the motion of the particle when $t > 0$.

5. A charge q of mass m and charge q is subject to a Newtonian frictional force

$$= - \gamma \vec{v},$$

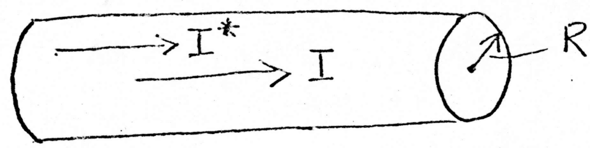
where γ = constant, $\vec{v}$ = instantaneous velocity. Upon it is incident a circularly polarized electromagnetic plane wave of intensity I and angular frequency ω in the z direction.

a. Calculate (in the steady state) the rate at which energy is absorbed from the wave by the particle.

b. Calculate directly (i.e., by use of the Lorentz force equation) the rate at which angular momentum is absorbed.

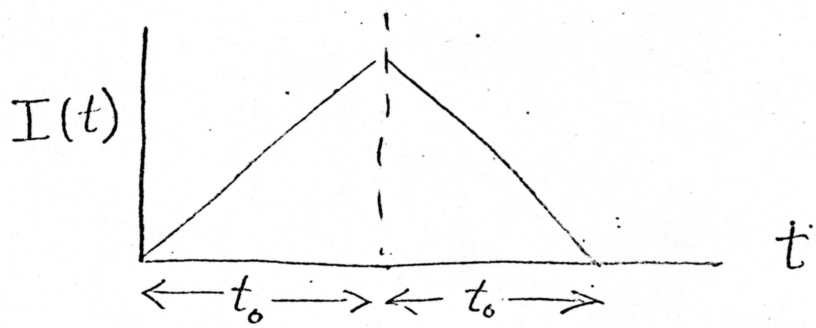
6. A current I(t) with uniformly distributed current density is passing through a very long cylinder of radius R, along the axis. The cylinder contains a gas which suddenly at some arbitrary time T becomes highly ionized, so that it acquires a large conductivity σ. After T no net charge exists inside the cylinder, and only the axial field E_z remains. The ionized gas obeys Ohm's law, $j^* = \sigma E_z$. Assume the

current density in the gas, j*, is uniform to the wall and neglect any special effects from this wall.



a. what simple equation relates I(t) to the current in the ionized gas, I*, for t > T?

b. Assume I(t) is triangular in time, i.e.,



T may fall anywhere in the lifetime of I(t). Consider two cases, $T = t_o/10$ and $T = t_o$. Solve your equation for the total current, I + I*, and sketch the solution. If time is pressing, use your intuition to sketch the result without solving the equation in detail.

Physics Qualifying Exam UCI October 4, 1972

QUANTUM MECHANICS II

1. a. Consider the bound state of an electron and a positron (positronium). Give an energy level diagram of this object through n = 2, labeling each state spectroscopically, giving the charge conjugation quantum number and parity.

   b. For n = 1, give the fine structure and Zeeman effect splittings by using an $A\,\vec{S}_1 \cdot \vec{S}_2$ interaction between the electron and positron.

2. Consider the helium atom. Estimate the ground state energy either by using perturbation theory or by the variational method.

   Potentially useful integrals and formulas.

$$\int d^3r\; e^{-2kr} = \frac{\pi}{k^3}$$

$$\int d^3r\; \frac{e^{-2kr}}{r} = \frac{\pi}{k^2}$$

$$\int d^3r_1 d^3r_2 \frac{e^{-2k(r_1 + r_2)}}{|\vec{r}_1 - \vec{r}_2|} = \frac{5}{8}\frac{\pi^2}{k^5},$$

$$\nabla_r^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

3. a. In atomic physics, the spin orbit interaction has the form $A\,\vec{\ell} \cdot \vec{s}$. Derive an expression for the coefficient A in a hydrogenic atom. Don't worry about factors of two, feel free to use qualitative arguments, but get the dependence on the relevant variables correct. Estimate the magnitude of the fine structure splitting in the optical spectrum of sodium.

   b. An electron in a state of orbital angular momentum ℓ is placed in a weak magnetic field. Find an expression for the g-factor.

Physics Ph.D. Qualifying Exam UCI October 5, 1972

THERMODYNAMICS AND STATISTICAL MECHANICS

1. A magnetic material is placed in an external magnetic field H. The external field is changed in an adiabatic manner by the amount ΔH.

   a. Show that the change in temperature of the system is given by

$$\Delta T = - \frac{T}{C_H(T)} \left(\frac{\partial M}{\partial T}\right)_H \Delta H ,$$

   where M is the magnetization, and $C_H(T)$ the specific heat in constant field. (Hint: Consider the use of the Maxwell relations for the magnetic system).

   b. For a paramagnetic system, $M = \chi H$. From what you know of the temperature dependence of χ for paramagnetic systems, do you expect $\Delta H > 0$? Do not just state the answer, but explain briefly.

2. A gas is described by the equation of state

$$P(V-b) = RT \exp\left[- \frac{a}{RTV}\right] ,$$

   where R is the gas constant. Sketch the form of the isotherms in the P-V plane, discuss briefly the behavior of the system in the interesting regions of the sketch, and find an expression for any critical points that may be present.

3. A set of N independent spins are placed in an external magnetic field H. The spins are at temperature T.

a. Find an expression for the entropy as a function of magnetic field and temperature.

b. Draw a sketch of the entropy as a function of magnetic field, for fixed temperature. Explain the physical origin of the value of the entropy as $H \to 0$, $H \to \infty$.

4. The electron density in a white dwarf star is 10^{30} electrons/CM^{-3}, and the temperature of the star is 10^7K. The electrons in the star move with speeds near the velocity of light so the energy of an electron is $\epsilon = cp$, where p is the electron's momentum. Assuming that this relation is valid for all p, compute the electron pressure P in the star. (Hint: the electrons form a degenerate Fermi gas at these temperatures and pressures.)

5. Two infinite parallel plates possess black-body surfaces, and are maintained at temperatures T_1 and T_2 respectively. Calculate the rate $q_{12}(\omega)$ at which energy of frequency ω is transferred from one surface to the other per unit area and unit time. Calculate the total rate of energy transfer when T_1 is close to T_2.

6. By the methods of statistical mechanics, show that the entropy of the ideal gas is given by

$$S = Nk\ So + Nk\ \ln(VT^{3/2}).$$

Exhibit an explicit expression for the constant So.

MATHEMATICAL PHYSICS

1. Minimize the function

$$f(x,y,z) = x^2 + 3xy + 2y^2 + 4yz + z^2$$

subject to the constraints

$$x + y = 1$$

$$zy + z = 1$$

2. Evaluate the integral

$$I(\alpha) = \text{P.V.} \int_0^\infty \frac{x^\alpha \, dx}{x^2-1} , \qquad 0 < \alpha < 1 .$$

3. Solve the homogeneous integral equation

$$\phi(x) = \lambda \int_{-\pi}^{+\pi} \cos(x-y)\, \phi(y)\, dy .$$

4. Using the saddle point method, obtain the leading term in the asymptotic behavior of the function I(k):

$$I(k) = \int_c e^{-t} \left(\frac{t + \frac{i}{2}}{t - \frac{i}{2}} \right)^{i \frac{k^2}{4}} dt .$$

5. Use the WKB approximation to discuss the energy levels of a particle of mass m moving in a one dimensional potential $V(x) = \frac{1}{2} k|x|^n$, where $n > 0$. In particular, what are the energy levels in the limit $n \to \infty$? Why is your answer reasonable?

6. Radioactive gas atoms are introduced into a non-radioactive background of the same gas, at the same temperature and pressure. The radioactive particles are introduced at $x = 0$ and $t = 0$ into a very long, narrow box, so a one dimensional diffusion process occurs.

a. The box walls are infinitely far away, and the probability distribution of particle position satisfies

$$\frac{\partial P(x,t)}{\partial t} = D \frac{\partial^2 P(x,t)}{\partial x^2}$$

b. Now suppose one wall of the box is brought to $x = L$, the other wall still at infinity. This wall at L absorbs all radioactive gas particles which reach it. What is $P(x,t)$ now? State your reasoning.

7. Determine the behavior of the function

$$\Gamma(Z) = \int_0^\infty e^{-t}\, t^{z-1}\, dt \quad , \quad z > 0$$

in the limit as $z \to 0$.

PHYSICS QUALIFYING EXAM UCI October 6, 1972

GENERAL PHYSICS SECTION

1. Order the following gases in order of increasing index of refraction:

$$S,\ HC\ell,\ He,\ CH_4,\ CH_3OH$$

2. What is the minimum energy in the center of mass frame necessary to produce a proton-anti proton pair? Derive the result.

3. Determine which of the following decays can proceed, and specify which interaction. If the decay cannot occur, explain the reason.

$$\Lambda^o \rightarrow p + e^- + \bar{\nu}_e$$

$$\pi^o \rightarrow \gamma + \gamma + \gamma$$

$$\Delta^+ \rightarrow n + \pi^+$$

$$K_L^o \rightarrow \pi^+ + \pi^-$$

4. (a) Can one do X-ray astronomy on the earth's surface? If not, explain why. If one can, explain briefly what kind of apparatus is required.

   (b) There has been a considerable amount of interest in the possibility of observing solar neutrinos. Why would this be interesting?

5. What wavelength radiation is required to make a photoelectric cell using potassium? Explain the reason for your answer.

6. In the late nineteenth century, the notions of quantum theory were developed in only primitive form, of course. As a consequence, physcists faced many puzzles. One of these was the very large electrical conductivity observed in metals

6. (cont'd.)

at low temperature (think of normal metals, not superconductors). Why was this a problem in those days, and what idea cleared up the problem?

7. At one time, physicists thought that electrons might be present inside the nucleus. From what you know of the properties of particles and their interactions, give one or more reasons why this possibility is unreasonable. Include order of magnitude estimates of the relevant physical quantities that enter into your considerations.

8. (a) Theoretical physicists postulated the existence of the neutrino many years before its existence was confirmed by experimentalists (Reines-Cowan experiment). What observation prompted Pauli to propose the existence of the neutrino?

(b) It is now realized that there are two distinct classes of neutrinos in nature. Briefly describe them, and in particular indicate the kinds of interactions in which each class of neutrinos participates.

9. On a clear night, signals from radio stations can be heard many thousands of miles from the source. On the other hand, television signals from transmitters comparable in power to radio transmitters can be received within only a few tens of miles of the antenna. Why? (Compare television signals with AM radio signals.)

10. The electrical conductivity of matter is a remarkable property, in that it may vary by as much as seventeen orders of magnitude, if one compares the conductivity of wide classes of materials with each other. Briefly and clearly explain the physical origin of the difference between metals,

10. (cont'd.)
insulators, and semiconductors. Crudely sketch the temperature dependence of the conductivity of

(a) a metal

(b) a semiconductor.

11. Below are listed (vertically) conservation laws, and horizontally types of particle interactions. Which laws are valid for which interactions? Put an X in the appropriate box if the law is invalid.

Parity	Strong	Electromagnetic	Weak
Charge Conjugation			
Baryon Conservation			
Lepton Conservation			
Time Reversal			
TCP			
CP			
Isotopic Spin			
Strangeness			

12. What is the approximate radius of

(1) proton
(2) electron
(3) Oxygen atom
(4) Earth
(5) Earth's orbit
(6) particle in fog
(7) Galaxy

13. Provide approximate estimates of

1. Surface temperature of sun.
2. Highest steady magnetic field which can be reached with superconducting magnets.
3. Earth's magnetic field at 5 earth radii.
4. Velocity of 100 Kev electron

13. (cont'd.)

    5. Lowest temperature currently attained.
    6. Scaling length (e - folding length) of Earth's atmosphere.
    7. Age of earth.
    8. Half life of neutron.

14. Plasmas exist in both metals and as hot, ionized gases. Describe briefly how these two types of plasmas differ, how the relevant parameters vary in magnitude and what sort of phenomena might be seen in one case, but not the other.

15. In the famous elevator gedanken-experiment with which Einstein illustrated the principle of equivalence, an observer could not observe a gravitational field <u>if</u> the elevator was freely falling. However, the earth is freely falling in the moon's gravitational field. Explain the tides.

16. A dust particle is placed inside a long pipe filled with air. If the particle strikes the walls it is simply reflected, and thus can move only along the pipe. Ten seconds later, the experimenter finds that the dust particle has moved one centimeter to the left. How long will it take the particle to move a meter from that position?

QUANTUM MECHANICS I

1. A one dimensional harmonic oscillator of charge e is perturbed by a weak electric field of strength E in the positive X direction.
   Calculate using perturbation methods the energy shift of each level to second order.
   Note: Use raising and lowering operator methods to evaluate matrix elements.

2. Angular Momentum and Spin-Statistics
   a. For a valence shell with two p electrons, write down the possible spectroscopic terms and their degeneracy.

   b. Write down an explicit, antisymmetric wave function in terms of simple particle wave functions for each state of the D term.

3. Consider the potential $V(x) = A\delta(x)$ $-a<x<a$ and

   $$V(x) = +\infty, \quad |x|>a.$$

   a. Find the eigenvalue condition and wave functions.
   b. Show graphically the eigenvalue equation solutions for the case $A > 0$.
   c. What do the eigenvalues approach as $A \to \infty$? What is the physical interpretation?

4. To a first approximation, the potential that a charged particle feels from a hydrogen atom can be thought of as that due to a positive point charge at the origin plus a uniform cloud of negative charge occupying a sphere of radius a_o about the origin.
   a. Calculate in the Born approximation the scattering amplitude for the scattering of a charged particle from this charge distribution.

(over)

4. b. Calculate the large momentum transfer limit of the amplitude and compare it to the pure Coulomb amplitude.

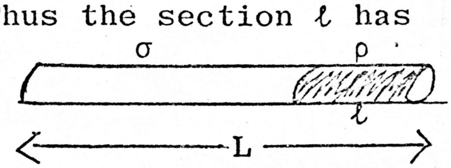
   c. Show that the total elastic cross section from this potential is finite.

5. Calculate the spontaneous transition probability for dipole transitions from <u>all</u> $n = 2$ hydrogen atom levels to the ground state.

CLASSICAL MECHANICS

1. A particle moves on a smooth circular wire under the action of a force inversely proportional to the n^{th} power of the distance from, and directed toward, a point on the wire. Find the value of n such that the force exerted by the wire on the particle at the point of contact is constant in time.

2. A stick is made so that part of its total length L is more dense. Thus the section ℓ has



density ρ and the rest of the rod has density σ. Both ρ and σ are in units in which water has a density of one. Now we throw this stick in a bathtub. Show that in order for the stick to float perfectly vertically we must have

(a) $X \equiv \sigma(L-\ell) + (\rho-1)\ell \leq L - \ell$

(b) $X^2 \geq \sigma(L-\ell)^2 - (\rho-1)\ell^2$

3. Consider a mass m connected to a spring with force constant k, under conditions where gravitational forces may be neglected. The mass undergoes harmonic oscillations with amplitude A. Suppose the force constant of the spring actually varies in time very slowly, i.e., the relative change in k in one cycle is << 1. How does the amplitude of oscillation behave?

(over)

4. A particle of mass m is suspended at rest from a point A by an elastic string of negligible mass and force constant k, whose initial unstretched length is a. At time $t = 0$ the point A begins to oscillate up and down so that its upward displacement at time t is $x_o \sin \omega t$. Find the length of the string at time t.

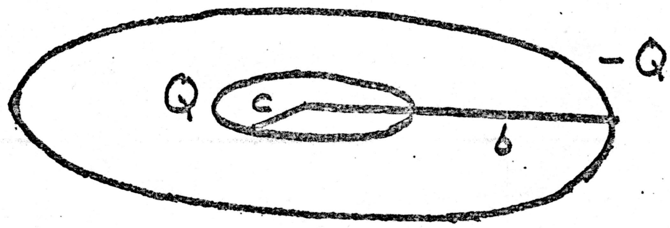
5. A smooth tube of mass M is bent into a circular helix of radius a and pitch $2\pi b$. The helix is mounted so as to rotate freely about its axis, which is vertical. A particle of mass m is dropped into the tube at the top when the tube is at rest and slides down the tube. Find the angular velocity of the helix and the vertical distance through which the particle falls as functions of t.

6. (a) Find the ratio of times required to traverse the same path for two particles having different masses but the same potential energy.

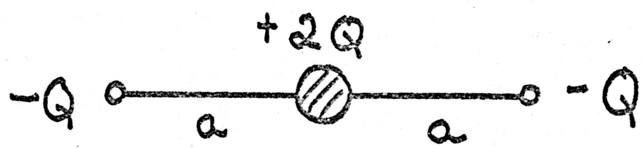
   (b) Find the ratio of times required to traverse the same path for two particles of the same mass but potential energies differing by a constant factor.

ELECTRICITY AND MAGNETISM

1. We begin with a ring charge $-Q$ of radius b, concentric and coplanar with a smaller ring charge Q of radius c.



We want to model this approximately at large distances as a linear quadrapole which looks like:



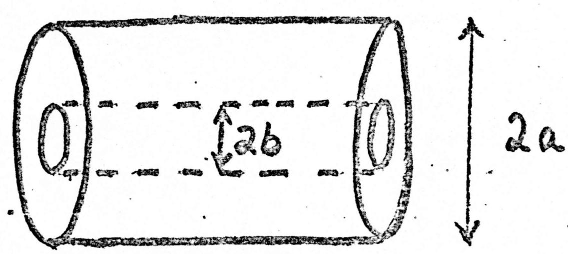
What should we take a to be in terms of b and c?

2. A certain plasma consists of a gas of free electrons, neutralized by a uniform positive background charge. The plasma is in a DC magnetic field, B, which is in the z-direction. Consider the propagation of low-frequency electromagnetic waves parallel to B in the plasma. (Hint: such waves are circularly polarized and purely transverse.) Show that for one sign of circular polarization the waves propagate, and for the other sign are attenuated. The propagating waves are called whistlers (in ionospheric physics) or helicons (in solid state physics). What is their dispersion relation? What is their group velocity?

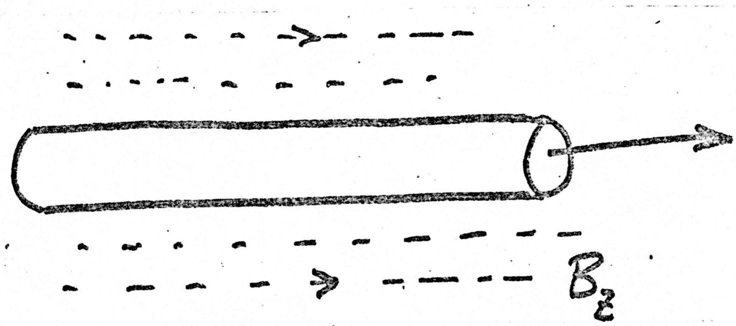
(over)

3. Derive an approximate formula for the energy loss of a fast charged particle traveling through matter, so as to display explicitly the principal dependence of the energy loss on velocity, charge, and mass of the particle.

4. A coaxial cavity with perfectly conducting walls and end plates is excited in its lowest mode. In terms of W, the energy stored in the electromagnetic field, determine the time-averaged pressure exerted by the field on the side walls and end walls.



5. An infinitely long cyclindrical current distribution, uniform to radius a, produces its own self-field B_θ as it propagates along an imposed homogeneous field B_z.



The electrons have a net charge density per unit length Q. The electrons have some perpendicular motion, $v_\perp << v_z$. Also, $B_\theta << B_z$. To first order in $v_\perp/v_z$ and B_θ/B_z, find the frequencies of the electron motion. What sort of orbits do these frequencies represent?

6. A collisionless electron gas in a magnetic field is neutralized by a fixed array of ions. We wish to propagate electrostatic waves through the electrons with $\omega >> kv_{thermal}$. Take the magnetic field effectively infinite, so the electrons move only in straight lines along the field (cyclotron radius is zero). Using Newton's laws (or the collisionless Boltzmann (Vlasov) equation) and Poisson's equation, calculate the dispersion relation of such waves, and comment. How would you include weak collisions with neutral atoms (impurities)?

Physics Qualifying Exam UCI September 25, 1973

THERMODYNAMICS AND STATISTICAL MECHANICS

1. The potential energy V or a linear chain of (N + 1) atoms is given by

$$V = \sum_{n=1}^{N} \{\tfrac{1}{2}A(U_n - U_{n-1})^2 + B(U_n - U_{n-1})^3 + C(U_n - U_{n-1})^4\}$$

where U_n is the displacement of the n^{th} atom from its equilibrium.

   a. Show that the cubic anharmonic term causes thermal expansion of the chain, and obtain the thermal expansion coefficient. Assume B and C are small compared with A.

   b. What is the heat capacity of the chain at temperature T.

2. A classical gas consists of diatomic molecules whose atoms interact through a harmonic force. Neglect interactions between atoms in different molecules. Let m be the mass of the atoms and ω the frequency of vibration of the molecule.

   a. Using the microcanonical ensemble, find the pressure and entropy as a function of temperature and volume and number of molecules.

   b. Do the same using the canonical ensemble.

3. A plane wave advances with velocity v into a stationary fluid whose pressure P and density d are everywhere constant. Behind the wave, the pressure and density are P' and d' and the fluid moves with velocity u; these quantities are everywhere constant.

   a. Show that the conservation of matter is expressed by the equation

$$\frac{d}{d'} + \frac{u}{v} = 1$$

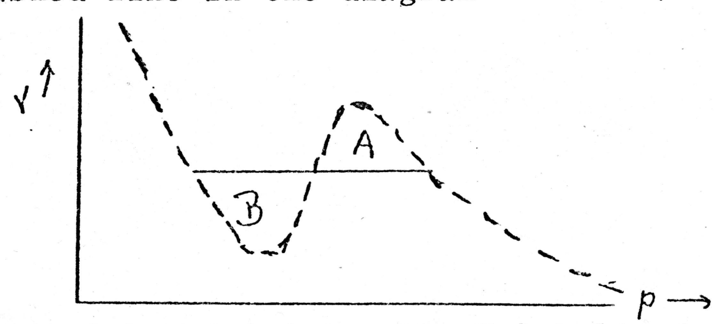
b. Show that conservation of momentum is given by

$$P' = P + duv$$

c. Now assume the fluid is an ideal gas, $P = RdT$, and has a specific heat $C_v = \frac{R}{\gamma-1}$. Show that conservation of energy is expressed by

$$\frac{RT'}{\gamma-1} = \frac{u^2}{2} + RT\left(\frac{1}{\gamma-1} + \frac{u}{v}\right)$$

4. The isotherm for a van der Waals gas has a shape as given by the dashed line in the diagram



Show that by minimizing the Helmholtz free energy we obtain the Maxwell construction, i.e., the true isotherm is the one given by the straight line with the areas of A an B equal.

5. A cylinder of radius R and length L rotates about its axis with constant angular velocity ω. It contains N particles. Evaluate the density distribution of an ideal gas enclosed in the cylinder. Ignore gravity. Do a classical calculation, assuming the thermal equilibrium is established at temperature T. (Hint: You can either try to guess the distribution function and carry on from there, or you can proceed through the usual statistical mechanical machinery. Note that the Hamiltonian describing the motion is $H^* = H - \omega L$, where H is the Hamiltonian in the coordinate system at rest and L is the angular momentum.)

6. Use the barometric formula

$$\rho = \rho_o\, e^{-\frac{Mg\,z}{RT}} \qquad (M = \text{molecular weight}),$$

to show that the heat capacity of an unbounded gas column under gravitational field equals C_p. Express the answer in terms of the mass of the air column.

QUANTUM MECHANICS II

1. An electron beam is diffracted around an infinitely long magnetized iron whisker. Ignoring for the moment the fact that it is magnetized, explain qualitatively what kind of diffraction pattern you expect to see. Now include the fact that the whisker is magnetized. How does the pattern change? Calculate the effect.

2. <u>Qualitative</u> - <u>Atomic Physics</u>

   a. Outline the interactions, approximations and conserved quantum numbers that yield the various stages of classification of the states of an atom. That is, explain the classification scheme of atomic physics, beginning with the central field approximation and working down to the spin-orbit coupling. Give the order of magnitude of the energy splitting at each stage.

   b. Show the Hamiltonian for the interaction with an external magnetic field. What levels are split by a weak magnetic field? What are an appropriate set of state labels or quantum numbers for a strong magnetic field.

3. Obtain the electric polarizability of a hydrogen atom in its ground state by calculating the change in the ground state energy, using a perturbation theory. Note that the hydrogenic wave function for the ground state is given by:

$$\psi_o = (\pi a_o^3)^{-\frac{1}{2}} \exp(-r/a_o) \quad \text{where } a_o \text{ is the Bohr radius.}$$

To simplify the problem, assume that all the excited states lie at E = 0, although their energies are actually given by

$$E_n = -\frac{1}{n^2} \cdot \frac{e^2}{2a_o}$$

where n is the principal quantum number. Does this approximation overestimate or underestimate the polarizability? (The polarizability α is obtained from the second order perturbation energy by $W_2 = -\frac{1}{2}\alpha\mathcal{E}^2$ where $\mathcal{E}$ is the electric field strength.)

GENERAL PHYSICS

1. The attractive or repulsive nature of a force cannot be determined from differential scattering cross section measurements ($d\sigma/d\Omega$). Why? Illustrate your explanation with a sketch.

2. What are the possible (electromagnetic) multipole moments $^{14}_{7}N_7$? (ground state spin-parity, 1^+). Why?

3. Consider the following reactions

   a) $\gamma + p \rightarrow p + \pi^o$

   b) $\gamma + p \rightarrow p + p + \bar{p}$

   What is the threshold photon energy for each reaction in terms of pion mass and proton mass?

4. What is the energy released when a negative muon goes from a state of n = 6 to n = 5 in lead? (Z = 82)

5. What is "coherent light"? How do you measure "coherence"?

6. When you see a piece of metal, you can generally tell that it is a metal. Discuss why you can tell a metal from other forms of solids. Are there non-metals which can look like a metal?

7. Explain with short arguments and sketches:

   a. Why is the sky blue?

   b. Why is the setting sun red?

   c. Why does the sun sometimes appear green just as it sets (the green flash)?

   d. Why do things appear colorless in moonlight?

   Two of these questions rely upon both physical and physiological effects.

(Over)

8. A. Olbers' Paradox (1826) states that if (a) the universe is infinite (and Euclidean), and (b) is uniformly populated with stars with some constant average absolute luminosity, then, ignoring the screening due to stars intercepting light travelling in space, the radiant energy flux at any point in space (say on the surface of the earth) is infinite. Prove this assertion.

B. If we take into account the screening effect ignored in A, then one can show that the energy flux (per unit area per unit time) at any point in space is the same as the average energy flux at the surface of a star. Prove this assertion.

9. Consider the elastic scattering process

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

for an incident anti neutrino on an electron. Derive the equation relating initial and final neutrino energies and the scattering angle.

10. Are negative temperatures realizable? If not, why? If so, describe such a physical system.

11. What is the "Island of stability" in nuclear physics? Why might it exist?

12. A. In a glass of water at $0^{\circ}C$, there floats a cube of ice. Enough heat is added to melt the ice at $0^{\circ}C$. Does the water level rise, fall or remain constant? Why? The water temperature is raised to $2^{\circ}C$, Does the water level rise, fall or remain constant?

B. Do winds in the northern hemisphere blow clockwise or counterclockwise around a center of low pressure? Why?

13. Compare the ground state energy of a particle in a one dimensional infinite square well with that of a finite square well. Discuss the differences, if any.

14. Plasmas exist in metals and as hot, ionized gases. Describe briefly how these two types of plasmas differ, how the relevant parameters vary in magnitude and what sort of phenomena might be seen in one case, but not the other.

MATHEMATICAL PHYSICS

1. Solve by Fourier transformation

$$(\nabla^2 - \frac{1}{L^2})\psi(\vec{x}) = A\delta(\vec{x})$$

Subject to the boundary conditions

$$\psi(\vec{x}),\ \nabla\psi(\vec{x}) \text{ vanish as } |\vec{x}| \to \infty .$$

2. a. Discuss the analytic properties of the function

$$F(z) = \int_{-\infty}^{\infty} \frac{e^{-x^2}dx}{x-z}$$

where the integration is along the real x-axis.

b. Consider a different function ($G(z) = F(z)$ for $\mathrm{Re}\, z > 0$. For $\mathrm{Re}\, z < 0$, $G(z)$ is defined by analytic continuation. How is this accomplished and how do the analytic properties of $G(z)$ differ from $F(z)$?

3. In a spherical sample of uranium 235 the neutron density $n(\underset{\sim}{x},t)$ obeys the differential equation

$$\nabla^2 n + n = \frac{\partial n}{\partial t} ,$$

with the condition $n = 0$ at the surface. Assuming that n is independent of the polar and azimuthal angles θ and φ, what is the smallest value of the radius a of the sphere for which n will erupt exponentially with time?

(over)

4. The electronic polarizability of a medium at frequency ω is

$$\alpha(\omega) = \alpha'(\omega) - i\alpha''(\omega)$$

Given that

$$\alpha''(\omega) = \frac{e^2}{m} \sum_{i=1}^{N} \frac{\Gamma_i \, \omega}{(\omega^2-\omega_i^2)^2+(\Gamma_i\omega)^2}$$

where ω_i, Γ_i, and e^2/m are constants. Also $\omega_i >> \Gamma_i$. Find $\alpha'(\omega)$ and discuss the physical and mathematical basis of the derivation.

5. Assume that A is a Hermitian Matrix.

a. Show that $(A + iI)^{-1}(A-iI)$ is unitary where I is the unit matrix.

b. If A has the eigenvalues $(\lambda_1\lambda_2\lambda_3\cdots\lambda_n)$ what are the eigenvalues of $(A + iI)^{-1}(A-iI)$?

6. Consider the irreducible representations of the rotation group $D^{(j)}$.

a. What are the characters of these representations?

b. Show how to reduce the product representation $D^{(3)} \times D^{(2)}$ by the method of characters.

7. By contour integration evaluate the integral

$$I(\alpha) = \text{P.V.} \int_0^\infty \frac{x^\alpha dx}{x^2-1} \quad \text{where } 0 < \alpha < 1.$$

Physics Qualifying Exam UCI May 20, 1974

(4 points) 1. A man standing on the edge of a cliff at some height above the ground below throws one ball straight up with initial speed u and then throws another ball straight down with the same initial speed. Which ball, if either, has the larger speed when it hits the ground? Neglect air resistnace.

(4) 2. A solid wooden sphere rolls down two different inclined planes of the same height but different inclines. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one incline than the other? If so, which one and why?

(4) 3. Under what sets of conditions is $\bar{N} = \frac{d\bar{L}}{dt}$ valid? ($\bar{N}$ is the total torque on a system, and $\bar{L}$ is its total angular momentum.)

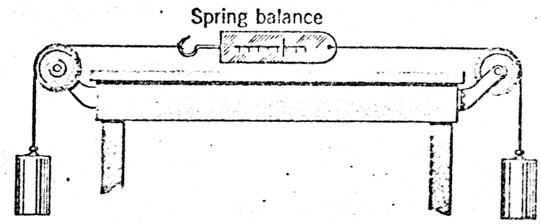
(4) 4. Define "virtual displacement," as used in the Lagrangian formulation of classical mechanics.

(4) 5. Why do the (holonomic) forces of constraint not appear explicitly in the Lagrangian equations of motion?

(4) 6. A satellite is in uniform circular motion about the earth. How much work is done on it during each revolution if its mass is m, its speed is v, and the radius of its orbit is R?

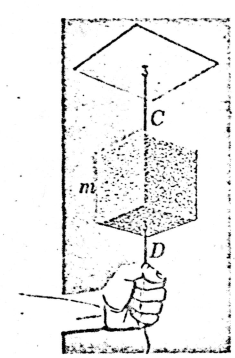
(4) 7. Consider a one-dimensional elastic collision between a given incoming body A and a body B initially at rest. How would you choose the mass of B, in comparison to the mass of A, in order that B should recoil with (a) the greatest momentum, (b) the greatest kinetic energy, and (c) the greatest speed?

8. Two 5 lb weights are attached to a spring scale as shown in
(4) the Figure. Does the scale read 0 lb, 5 lb, 10 lb, or give some other reading?



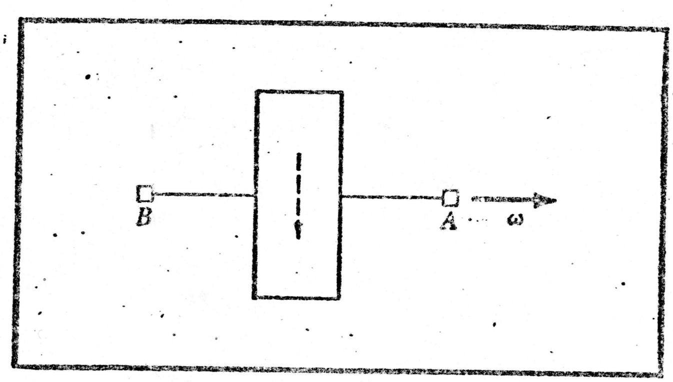
9. Can a sailboat be propelled by air blown at the sails from
(4) a fan attached to the boat?

10. A block of mass m is supported by a cord C from the ceiling,
(4) and another cord D is attached to the bottom of the block. Explain this: If you give a sudden jerk to D it will break, but if you pull on D steadily, C will break.



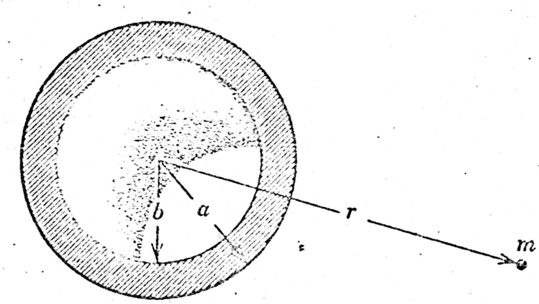
(4) 11. A ball is rolling (without slipping) on a horizontal plane. It then rolls (without slipping) up an inclined plane. (There must of course be friction present in order for it to roll without slipping.) Now suppose that the above inclined plane is replaced by an identical frictionless one (horizontal plane remains unchanged). Compared to the plane with friction, will the ball travel a greater, lesser, or the same distance up the frictionless plane before momentarily stopping and beginning to go back down the plane? Why? (Assume the initial state is the same in both cases, i.e. it is rolling without slipping on the horizontal plane.)

(4) 12. The figure represents a gyroscope wheel seen from one side, with its axis mounted in bearings A and B. It is spinning with angular velocity as shown, the near side of the wheel moving downward. Upward support forces exist equally at A and B. It is now desired to reorient the wheel to place A directly over B, without moving the center of mass of the system. Describe the additional forces to be applied at A and B.

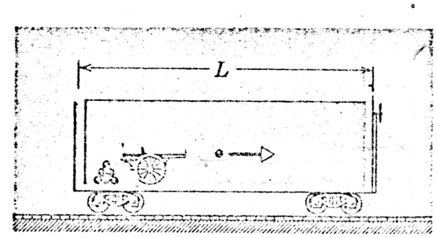


(6) 13. A ball of radius r and mass m rolls without slipping on the inside of a large hemispherical bowl of radius R. If the ball starts from rest at the top edge of the bowl, calculate the force that the bowl exerts on the ball at the bottom of the bowl.

(7) 14. A sphere of matter, radius a, has a concentric cavity, radius b, as shown in cross section in the figure. (a) Sketch the gravitational force F exerted by the sphere on a particle of mass m, located a distance from the center of the sphere, as a function of r in the range $0 \le r \le \infty$. Consider points $r = 0, b, a,$ and ∞ in particular. (b) Sketch the corresponding curve for the potential energy V(r) of the system.



(7) 15. A cannon and a supply of cannon balls are inside a sealed, impenetrable railroad car as in the Figure. The cannon fires to the right, the car recoiling to the left. The cannon balls remain in the car after hitting the far wall. Show that no matter how the cannon balls are fired the railroad car cannot travel more than its length L, assuming it starts from rest.



(7) 16. An hour glass is weighted on a sensitive balance, first when sand is dropping in a steady stream from the upper to the lower part and then again after the upper part is empty. Are the two weights the same? Explain.

(7) 17. Consider a uniform sphere of soft rubber (or jello).

(a) Write down its moment of inertia tensor (about its geometrical center) in matrix form, i.e.

$$I = k \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix},$$

where k is some overall constant which you need not determine.

(b) Suppose this sphere now rotates about a diameter (call it the z-axis). (It of course then changes its shape due to the "centrifugal" force. However, assume its density remains constant.) Write down its new moment of inertia (about its geometrical center). Clearly indicate how each element is related to every other element in the matrix. Also clearly indicate how each element is related to its corresponding one in part (a) above.

In each of the following problems:

(a) Find the Lagrangian

(b) Find the Hamiltonian

(c) List all holonomic and nonholonomic constraints

(d) State whether the external (i.e. nonconstraint) forces are conservative or not

(e) List the ignorable coordinates

(f) State whether or not H = T+V. Whether $\frac{dH}{dt} = 0$ or not.

(6) 18. A gas molecule moving in a cubical box with gravity acting.

(6) 19. A particle moving in the attractive central field with

$$\bar{F} = -(k/r^2)\, e^{-\alpha t}\, \hat{r} .$$

(6) 20. A bead sliding on a rough wire bent in the form of a helix (cylindrical coordinates: $z=a\theta$, $\rho=b$, a and b are constants and ρ, θ, z are the usual cylindrical coordinates.) The origin is a center of attractive force $\bar{F} = -kr^3\hat{r}$ where r is the distance from the bead to the origin.

Physics Qualifying Exam UCI May 20, 1974

QUANTUM MECHANICS I

1. a. Using the uncertainty relation, calculate the ground state energy of a hydrogen atom.

   b. Again using the uncertainty relation, calculate the ground state energy of a two electron atom whose nucleus has a charge Ze. Approximate the repulsive electron-electron interaction for electrons at $\bar{r}_1$ and $\bar{r}_2$ by $\dfrac{e^2}{r_1 + r_2}$ since the repulsion most likely places them on opposite sides of the nucleus.

2. a. Calculate the scattering amplitude in Born approximation for the gaussian potential

$$V(r) = V_o\, e^{-r^2/2r_o^2}$$

   b. Calculate the total elastic cross section for this potential.

   You may need:

$$\int_{-\infty}^{\infty} dx\, e^{-ax^2+ibx} = \sqrt{\frac{\pi}{a}}\, e^{-\frac{b^2}{4a}}$$

3. Consider two different one dimensional harmonic oscillators perturbed by a third "spring" depending on their displacement.

$$H = \frac{p_1^2}{2m} + \tfrac{1}{2}\, m\omega_1^2 x_1^2 + \frac{p_2^2}{2m} + \tfrac{1}{2}\, m\omega_2^2 x_2^2 + K'\, x_1 x_2$$

   a. Find the eigenstates of the unperturbed Hamiltonian, using raising and lowering operators to construct the states.

   b. Using the raising and lowering operator method, evaluate the energies using perturbation theory.

4. The atomic ground state of a muon orbiting about a nucleus is exactly like that of an electron in a single electron atom, with the substitution $m_e \rightarrow m_\mu = 106\ \text{MeV}/c^2$ in all formulae. However, because the radius of a muon's orbit is smaller than that of an electron, the finite extent of the nuclear charge distribution will produce a shift of the muon's ground state energy from its value for a point nucleus. Calculate this energy shift to first order in perturbation theory for a muon bound to an iron nucleus ($Z = 26$, radius $R \approx 4 \times 10^{-13}$ cm). Assume the iron nucleus to be a uniformly charged sphere, for which the electrostatic potential is

$$\varphi(r) = \frac{Ze}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right] \qquad r \leq R$$

5. The muon is a spin $\frac{1}{2}$ particle of mass $106\ \text{MeV}/c^2$, with the magnetic moment of a point Dirac particle ($g = 2$). Consider a negative muon captured in an atomic state by a nucleus. The increased mass of the muon relative to an electron will have an effect on all aspects of such a state. Make a quantitative comparison of the muon's atomic states with the corresponding electronic states, for each of the following aspects. In each case, give a physical explanation for the effect of the muon's great mass on that aspect (based on a Bohr model of the atom and your understanding of the relationship of magnetic moments, angular momenta, etc.). Note that the radius of a muon's orbit is smaller than that of an electron by a factor (m_e/m_μ).

a. principle level structure
b. fine structure (strength of spin-orbit coupling)
c. hyperfine structure
d. Zeeman splitting of levels in an external field.

6. A spin $\frac{1}{2}$ particle decays at rest into a spin $\frac{1}{2}$ and a spin 0 particle. The final state is a superposition of orbital angular momentum 0 and 1. (Parity is not conserved in the decay). The decay amplitudes leading to these orbital angular momentum states are a_s and a_p. Derive the expression for the angular distribution of the decay products in terms of a_s, a_p, and the polarization of the initial particle. Use the Clebsch-Gordan coefficients $C(J, M, j_1, m_1, j_2, m_2)$ and the spherical harmonics Y_ℓ^m :

$$Y_0^0 = \sqrt{1/4\pi} \qquad C(\tfrac{1}{2},\tfrac{1}{2},1,1,\tfrac{1}{2},-\tfrac{1}{2}) = \sqrt{2/3}$$

$$Y_1^0 = \sqrt{3/4\pi}\cos\theta \qquad C(\tfrac{1}{2},\tfrac{1}{2},1,0,\tfrac{1}{2},\tfrac{1}{2}) = \sqrt{1/3}$$

$$Y_1^{\pm 1} = \mp\sqrt{3/8\pi}\sin\theta\, e^{\pm i\phi}$$

Ph.D. Qualifying Exam UCI May 21, 1974

GENERAL PHYSICS

1. Describe in a sentence or two the physical principles behind 4 of the following::
   a. The Mossbauer effect
   b. A hologram
   c. Cerenkov radiation
   d. Synchrotron radiation
   e. Compton scattering
   f. Gravitational redshift
2. What is the laboratory experimental evidence that the rest mass of the neutrino is very small? Approximately what is the upper limit to the neutrino rest mass derived in this fashion?
3. Neutral potassium has a single s electron outside a closed shell. What are the L, S and J values of its ground state? What is the spectroscopic designation for such a term?
   What are the L, S and J values and term designation for the ground state of singly ionized potassium?
4. The uncertainty principle and the existence of the neutron together set an upper limit to the density of matter consisting of electrons and protons (or other nuclei) that can exist. Calculate this limiting density for matter consisting entirely of electrons and protons, making use of the uncertainty principle in the form $\Delta x \, \Delta p = \hbar$ (which tells you the typical energy that an electron must have as a function of density) and the fact that the mass of the neutron is about 1.3 MeV larger than the mass of the proton.

   Useful numbers: $h = 6.626 \times 10^{-27}$ erg sec

   $m_e = 9.11 \times 10^{-28}$ gram

   $1 \text{ eV} = 1.602 \times 10^{-12}$ ergs $=$

   1.78×10^{-33} grams

5. Calculate the range of the nuclear force, assumed to be carried by virtual mesons, on the basis of the uncertainty principle ($\Delta E \ \Delta t = \hbar$) and the mass of the meson (about 140 MeV/c^2).

How does this compare with the size of a typical nucleus?

6. A capacitor of capacitance C is in equilibrium with a heat bath at temperature T. Calculate the root mean square noise voltage across the capacitor.

7. A gas consists of atoms of mass M at temperature T and emits radiation at frequency ν. Compute the root mean square value of the Doppler line width. You may assume that the Doppler width is small compared to ν and that the atoms have velocities small compared to c.

8. The average density of matter in the universe is about 10^{-30} g cm^{-3}. Suppose all the matter were converted into black body radiation, which would be added to the present 3^{o} Kelvin black body radiation. Compute the resulting temperature.

$$U(\nu)d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

You may find the following integral useful:

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \pi^4/15$$

9. Give approximate numerical values <u>including units</u> for the following:

a. The mass of the proton
b. The magnetic field at the earth's surface
c. The age of the universe
d. The binding energy of the deuteron
e. The ionization potential of hydrogen in its ground state
f. The speed of light in a vacuum
g. The speed of light in a medium with index of refraction 1.5
h. The distance from the earth to the sun
i. The diameter of a hydrogen atom in its ground state
j. The Bohr magneton

10. Two rings rotate with equal and opposite angular velocity about a common center. Suppose Adam rides on one ring and Eve on the other, and that at some moment they pass each other and their clocks agree. At the moment they pass, Eve sees Adams clock running more slowly than her own, so she expects to be ahead the next time they meet. But Adam expects just the reverse.

    What really happens?

    What simple physical principle indicates that this must be so?

11. Associated with each of the standard conservation laws of physics is some type of transformation under which the laws of physics are invariant. What is the conserved quantity associated with the invariance of physical laws to:

    a. Rotations of the coordinate system

    b. Translations of the coordinate system

    c. Lorentz transformations

    d. Gauge invariance

12. State Kepler's three laws of motion. Use one of them to compute the distance of Jupiter from the sun, given that its orbital period is about 12 years.

13. Why does the earth's magnetic field deviate sharply from a dipole pattern?

14. List any two nuclear reactions responsible for producing energy in the sun.

15. A certain ensemble of nuclei has a mean lifetime τ. Sketch a neat curve for the decay rate as a function of time.

1. a. Find the complete wave function for the ground state of two electrons in a one dimensional box.

   b. Evaluate the energy due to a weak spin-spin interaction $V_o \bar{S}_1 \cdot \bar{S}_2$.

   c. Write down the lowest energy wave functions for three electrons in the one dimensional box.

2. a. Consider two 3 dimensional potential wells:

   a finite "square" well $\left(\begin{array}{ll} V = -V_o & r < R \\ V = 0 & r > R \end{array} \right.$

   and an infinite square well $\left(\begin{array}{ll} V = -V_o & r < R \\ V = +\infty & r > R \end{array} \right.$.

   In which well will the energies of the first bound state be higher? Why?.

   b. The $\ell = 0$ energy levels of a particle bound in a potential well have separations which <u>increase</u> with energy for an infinite square well potential, are constant for a harmonic oscillator potential, and <u>decrease</u> with energy for a Coulomb potential. Explain this behavior qualitatively in terms of the shapes of these three potentials.

3. The deuteron is known to have J = 1 and to be a bound state of a neutron and a proton which is predominantly L = 0 S = 1 It has a small positive quadrupole moment, implying that it is slightly cigar shaped, and not pure L = 0 S = 1.

   a. What combinations of L and S might form a J = 1 state?

   b. Of these, which will <u>not</u> be expected to mix with the L = 0 S = 1 state in the deuteron? Why?

c. The mixing of states with different L implies a noncentral np potential which might depend linearly on various combinations of L, S, and the relative positions and momenta of the nucleons, r and p. Which of the following dependances will not mix values of L or must otherwise be excluded? Give reasons.

$$\mathbf{r}\cdot\mathbf{S} \qquad (\mathbf{L}\cdot\mathbf{S})^2$$

$$(\mathbf{r}\cdot\mathbf{S})^2 \qquad \mathbf{p}\cdot\mathbf{S}$$

$$\mathbf{L}\cdot\mathbf{S}$$

d. Classically a spinning object will tend to bulge at the equator. How can a deuteron be cigar shaped?

4. Consider a free atom.

a. What are the selection rules on ℓ and m_ℓ for electric dipole radiation?

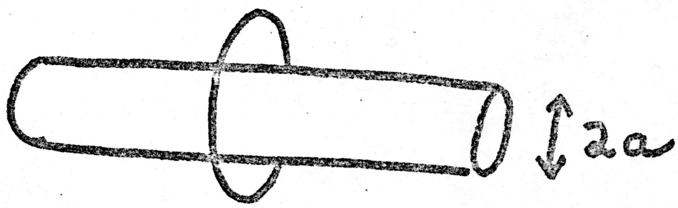
b. Why is one angular momentum coupling ruled out for dipole radiation?

c. What transitions are strictly forbidden?

d. Roughly by what factor is the $n = 3$, $\ell = 2$ decay directly to the ground state suppressed with respect to the $n = 3$, $\ell = 1$ decay to the ground state of a hydrogen atom.

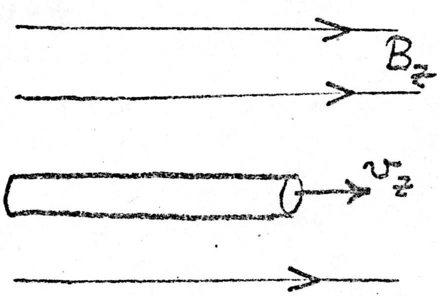
ELECTRICTY AND MAGNETISM

1. An infinite solenoid has n turns of wire per unit length and radius a. Outside this solenoid is a thin coil, of radius b, concentric with the solenoid and having N turns of wire in it. Find the mutual inductance between these two objects.



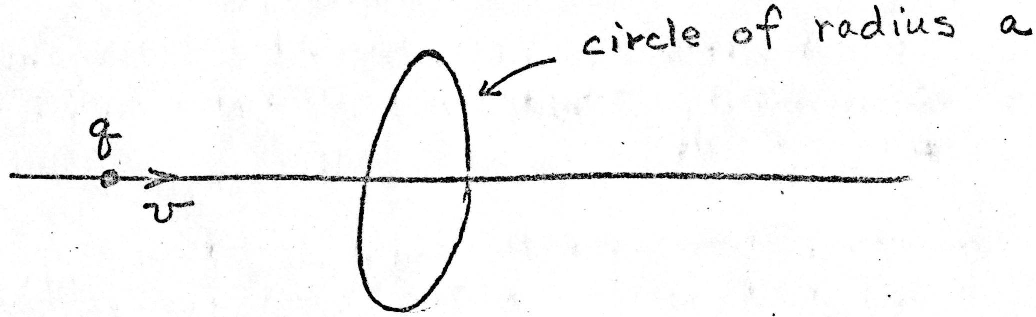
2. High energy X-rays (of frequency ω) propagate in vacuum until they strike a metal plate. The electrons inside are free (not bound to individual ions) and have density n. Find the critical angle of incidence for which the X-rays will be completely reflected by the plate.

3. An infinitely long cylindrical current distribution, uniform to radius a, produces its own self-field B_θ as it propagates along an imposed homogeneous field B_z. The electrons have a net charge density per unit length Q. The electrons have some perpendicular motion, $v_\perp \ll v_z$. Also, $B_\theta \ll B_z$.



To first order in $v_\perp/v_z$ and B_θ/B_z, find the frequencies of the electron motion. What sort of orbits do these frequencies represent?

4. A charge q moves with uniform velocity v (where v is very much less than c) along the axis of a circle C of radius a:



a. Calculate the magnetic field at C as a function of the time, t, using Ampère's law. Get $\oint \underline{B} \cdot d\underline{x}$ around C.

b. Let ψ = electric flux through C = $\int_S \underline{E} \cdot \underline{n}\, da$, where S = circular disc bounded by C. Calculate ψ as a function of t using Coulomb's law.

c. Verify $\frac{\partial \psi}{\partial t} = c^2 \oint \underline{B} \cdot d\underline{x}$.

(Note: Constant factor will depend on the units. M.k.s. units are used above.)

d. How does $\psi(t)$ change just as q crosses center of C? Explain why $\oint \underline{B} \cdot d\underline{x}$ is continuous anyway.

5. The most commonly accepted model of a pulsar assumes it is rotating at angular frequency Ω and is an excellent conductor inside, so

$$\underset{\sim}{E} \times \frac{v}{c} \times \underset{\sim}{B} = 0$$

inside. $\underset{\sim}{B}$ is constant and normal to the equatorial plane everywhere inside. Here $\underset{\sim}{v} = \underset{\sim}{\Omega} \times \underset{\sim}{r}$. Suppose outside this rotating sphere of radius R there is a vacuum.

a. Find the electrostatic potential for $r > R$.

b. From this, find the surface charge density.

c. Find the electric field at the surface. How does the electrical force compare with the gravitational force on an electron?

6. Consider a spherical planet (radius R) made of insulator. It rotates with angular frequency Ω and has gravitational acceleration g'. At an angle θ measured from the pole we place, infinitesimally above the planet, a small disk made of insulator, having mass m. Then we begin introducing equal charge densities σ uniformly on the surfaces of both the planet and the disk. At what value of charge density σ will the disk begin to rise off the planet? (Assume the disk rotates with the planet.)

Mathematical Physics

1. Find the Fourier transform of $\psi_n(x)$ given that

$$\psi_n''(x) + (2n + 1 - x^2)\psi_n(x) = 0$$

$$\psi_n(x) = 2^{-n/2}\,\pi^{-\frac{1}{4}}(n!)^{-\frac{1}{2}}e^{-x^2/2}\,H_n(x)$$

$$\int_{-\infty}^{\infty} \psi_n^2(x)\,dx = 1$$

$$\psi_n(-x) = (-1)^n\psi_n(x)$$

2. Evaluate the integral

$$\int_0^{\infty} \frac{\sin ay\, dy}{e^{2y} - 1}$$

3. Assume that f(x) is a well-behaved function and $\int_{-\infty}^{\infty} |f(x)|\,dx$ exists. Prove that if $F(x) = \frac{P}{\pi}\int_{-\infty}^{\infty} \frac{f(x')\,dx'}{x' - x}$, then

$$f(x) = \frac{-P}{\pi}\int_{-\infty}^{\infty} \frac{F(x')\,dx'}{x' - x}$$

(P stands for the principal part of the integral.)

4. Give examples of hyperbolic, elliptic and parabolic linear differential equations. Discuss what boundary conditions result in a unique stable solution in each case.

5. Prove that

$$\lim_{k\to\infty} \int_{-\infty}^{\infty} dx' \left\{\frac{2}{\pi} \frac{\sin^2[k(x-x')/2]}{k(x-x')^2}\right\} f(x') = f(x)$$

$$\text{or } \lim \frac{2}{\pi} \frac{\sin^2 kx/2}{kx^2} = \delta(x)$$

6. Take the potential function

$$V(x) = \infty \qquad x < 0$$
$$= ax \qquad x > 0$$

In the one-dimensional Schrodinger equation,

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) - E\right)\psi(x) = 0$$

you are to use one of the following trial functions to estimate the energy of the lowest bound state.

(1) Of the two choices,

(a) $\psi(x) = 2\beta^{3/2}\, x\, e^{-\beta x}, \qquad x > 0$

$\psi(x) = 0, \qquad x < 0$

(b) $\psi(x) = A\, x^2 e^{-c^2 x^2} \qquad$ (all x)

which do you think is the best initial choice?

5. (Continued)

(2) Find an approximate value of the ground state energy, using your choice from part (1). (Assume $\psi(x)$ is already normalized to unity.)

7. Consider a molecule which is in the shape of a tetrahedron (and has "T" as its symmetry group). Every electron in this molecule must of course also have this same symmetry, i.e., all electron wave functions must be basis functions for some irreducible representation of the group T. Suppose we apply a uniform magnetic field (Zeeman effect) parallel to one of the 3-fold symmetry axes. Show how each molecular electronic level splits.

$$\omega = 2\pi i/3, \quad \varepsilon = 2\pi i/6.$$

23 T	E	C_{2m}	C_{3j}^{+}	C_{3j}^{-}
A $x^2+y^2+z^2$	1	1	1	1
$E\,(x^2-y^2, (2z^2-x^2-y^2)/\sqrt{3})$	1	1	ω	ω^*
	1	1	ω^*	ω
$T\,(x, y, z), (I_x, I_y, I_z), (yz, zx, xy)$	3	-1	0	0

3 C_3	E	C_3^{+}	C_3^{-}
A z, I_z	1	1	1
$E\,(x, y), (I_x, I_y)$	1	ω	ω^*
	1	ω^*	ω

$3m$ C_{3v}	E	$C_3^{\pm}$	σ_{dt}
A_1 x^2+y^2, z, z^2	1	1	1
A_2 I_z	1	1	-1
$E\,(x, y), (zx, zy), (I_x, I_y), \left(\frac{x^2-y^2}{\sqrt{2}}, xy\right)$	2	-1	0

$\bar{6}$ C_{3h}	E	S_3^{-}	C_3^{+}	σ_h	C_3^{-}	S_3^{+}
A' I_z	1	1	1	1	1	1
A'' z	1	-1	1	-1	1	-1
$E''\,(I_x, I_y)$	1	ε	ε^2	-1	ε^4	ε^5
	1	ε^5	ε^4	-1	ε^2	ε
$E'\,(x, y)$	1	ε^2	ε^4	1	ε^2	ε^4
	1	ε^4	ε^2	1	ε^4	ε^2

I. Consider a system of N_A particles of type A and N_B of type B. Particles B are <u>fixed</u> at lattice sites R_j; $j=1, \ldots N_B$ and those of type A are free to move. A force between an A particle at position r and a B particle at position R is due to a potential

$$v(\underset{\sim}{r}-\underset{\sim}{R})$$

whose range is less than one half the lattice separation between B particles. Find the equation of state for this system. Express your answer in terms of

$$f = \int \left[e^{-\beta v(\vec{r}-\vec{R})} - 1 \right] d\vec{r} \quad .$$

II. A substance has the following properties.

(i) At a constant temperature T_o the work done by it on expansion from V_o to V is

$$W = RT_o \ln \quad (V/V_o) .$$

(ii) The entropy is given by

$$S = R \frac{V_o}{V} \left(e^{T/T_o} - 1\right)$$

where V_o, T_o are fixed constants.

(a) Calculate the Helmholtz free energy

(b) Find the equation of state

(c) Find the work done at an arbitrary temperature T.

III. a. Obtain the pressure and density of an ideal Bose-Einstein gas in two dimensions. Express your answer in terms of the functions

$$g_n(z) = \frac{1}{\Gamma(n)} \int \frac{x^{n-1}dx}{z^{-1}e^x-1}$$

b. Does this system exhibit a Bose-Einstein condensation?

IV. Obtain the relation between the pressure and internal energy of an ultrarelativistic Boltzmann gas in d dimensions. Discuss the result.

V. Consider a system with a gap in its energy spectrum. Specifically let the ground state have $E_o=0$ and the next state's start at $E=\Delta$ where Δ is non-zero for all N and V. Show that the specific heat C_v goes to zero as

$$C_v \sim \text{const. } e^{-(\Delta/kT)}$$

as $T \to 0$. Use either Boltzmann or Bose statistics.

VI. Consider a system of N noninteracting particles each having two internal energy levels E_o and E_1. Find the specific heat at constant volume of this system. (Use Classical Stat. Mechanics).

PHYSICS QUALIFYING EXAM UCI September 23, 1974

CLASSICAL MECHANICS

(Do 4 of these problems) Two Hours

1. In search of more living space, an intelligent race plans to redistribute their planet's mass into a giant ring, centered on their star and spinning with angular frequency Ω. Their leading physicist argues that this design is stable. Evaluate this claim. (Drawing pictures is usually a clearer method of explanation.)

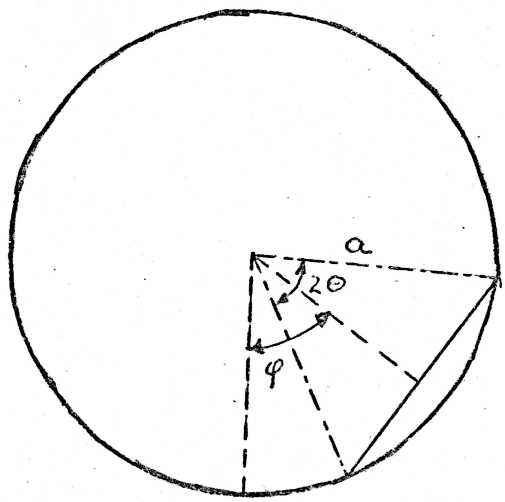
2. A cylinder of radius R is filled to a height y_o with water. Then it is set spinning with angular frequency Ω. Find the curve described by the surface of the water, and the height y of the water at the axis of the cylinder.

3. a) In the early 1960's physicists at UCSD proposed a cable be lowered from a synchronous satellite (whose orbital period equaled 24 hours, in the plane of the equator) all the way to the surface. Then packages could be put in orbit simply by running them up the cable on a pulley system, or something similar. Find the thickness of the cable as a function of height.

   b) We dig a straight tunnel through the earth at an angle less than 90^o to vertical, say between London and Moscow. A train then slides without friction back and forth between the cities. If the train moves freely, find the period of its motion.

4. A particle of mass m moves in a one-dimensional harmonic potential well $V(x) = \frac{1}{2} kx^2$. The parameter k varies slowly with time. What is the adiabatic invariant I(k) for this system?

5. A uniform rod in a uniform graviational field slides with its ends on a smooth vertical circle of radius a as shown in the figure. If the rod subtends an angle of $2\theta < 180^\circ$ at the center of the circle, find the frequency of small oscillations of the rod (small φ).



6. Which of the following coordinate transformations is canonical?

a) $Q = q^{\frac{1}{2}}\cos 2p \qquad P = q^{\frac{1}{2}}\sin 2p$

b) $Q = -p^2, \qquad P = -q^2$

c) $Q = \ln(p^2 + q^2), \quad P = \tan^{-1}\frac{q}{p}$

Physics Qualifying Examination UCI September 23, 1974

QUANTUM MECHANICS I

(Do 4 of these 5 problems. 3 hours.)

1. An electron moves in a crossed, (D.C.) static electric and magnetic field. Find the quantum mechanical energy levels of the electron. Take the electric field along the z axis, and the magnetic field along the x axis.

2. Given the following wave function,

$$\psi(r,\theta,\varphi) \propto r^3 e^{-Cr} \cos\theta \sin^2\theta e^{-2i\varphi}$$

Calculate:

a) The z component of angular momentum.

b) The total angular momentum.

c) The potential, $V(r)$.

d) The energy if $V(r) \xrightarrow[r \to \infty]{} 0$

3. An irreducible tensor operator $T(k,q)$ under the rotation group can be coupled with a representation of the group to form a new representation

$$|j_1 j_2 jm\rangle' = \sum_{m_1 m_2} T(j_1 m_1)|j_2 m_2\rangle\langle j_1 j_2 m_1 m_2|j_1 j_2 jm\rangle .$$

a) As a part of verifying that this new state is a representation, evaluate

$$J_z|j_1 j_2 jm\rangle'.$$

b) Using the identity $J_- J_+ = \bar{J}^2 - J_z^2 - \hbar J_z$

show that the norm of these states $'\langle jm|jm\rangle'$ is independent of m.

4. Consider the ground state of the one dimensional harmonic oscillator with the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda_o x^4$$

Use a variational method with the variable parameter α to bound the ground state energy.

Given:
$$\int_{-\infty}^{\infty} x^{2n} e^{-\alpha^2 x^2} = \begin{cases} \dfrac{\sqrt{\pi}}{\alpha} & n = 0 \\[2ex] \dfrac{1(3)(5)\cdots(2n-1)\sqrt{\pi}}{2^n \alpha^{2n+1}} & n = 1, 2, \cdots \end{cases}$$

5. A spin one particle is initially in the state $S_z = +\hbar$

a) What are the possible results of a measurement of S_x?

b) With what probability would they occur?

c) What is the expectation value of S_x?

d) If the measurement of S_x gives $S_x = +\hbar$, what would a successive measure of S_x give? Of S_z?

QUANTUM MECHANICS II

(Do 2 of these 4 problems. One hour)

1. A particle moves above a table (surface in the xy plane) in the gravitational potential mgz. Through use of the uncertainty principle, estimate the quantum mechanical ground state energy of the particle, and its average distance above the table. Estimate this average distance for an electron.

2. a) What are the spectroscopic terms (LSJ) of the lower energy states of the sodium atom (Z = 11).

   b) For a positive spin orbit coefficient, calculate the relative fine structure splitting and sketch the energy level diagram.

3. Describe in general physical terms under what conditions the following approximation methods for scattering would be used, and what throwing away higher order terms will neglect.

   a) Born Approximation

   b) Phase Shifts and Partial Waves

   c) Resonance Approximation

   d) Eikonal Approximation

   e) WKB

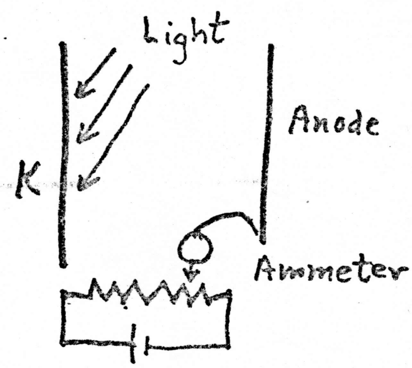
4. Derive the expression for the Hamiltonian in the presence of a weak uniform magnetic field to leading order.

Physics Qualifying Exam UCI September 24, 1974

GENERAL PHYSICS

(Do 10 of 18 problems) Two Hours

1. A surface of metallic potassium is illuminated with monochromatic of various wavelengths. In each case, the potential difference is found for which no electrons reach the anode. Two pieces of data are given:



Wavelength (Å)	2000	5000
Potential difference-volts	f.11	0.41

Find the value of Planck's constant.

2. The "saturation" field that can be obtained from magnetizing iron is about 20,000 gauss. Assuming that this field is produced by the alignment of a few electron spins per atom, estimate the spin-magnetic moment of the electron.

3. The flux of energy from the sun at the earth is about 2 calories per square centimeter per minute on an area perpendicular to the direction of incidence. Under the assumption (a very poor one, to be sure) that the incident radiation is monochromatic, estimate the amplitude of both B and E.

4. Hydrogen in the ground state is in a $2s_{1/2}$ state. The spin of the proton is 1/2 and the hyperfine structure splitting is about 1420 Mh_z. The proton magnetic dipole moment is about 2.8 nuclear magnetons. Estimate the effective magnetic field within the atom at the point where the proton exists.

5. Making use of the equivalence principle (gravitation equivalent to accelerated frame) derive an expression for the gravitational red shift. Is this measurable near the earth's surface?

6. Hydrogen gas in a laboratory discharge tube shows a discrete spectrum. At the same temperature and pressure, but in a massive system such as a star, it shows more nearly a black body radiation spectrum. Explain.

7. Can a spin-$\frac{1}{2}$ system have a permanent electric dipole moment if

(a) Parity is conserved?

(b) Parity is not conserved but time reversal invariance holds?

Why?

8. Can a gamma ray convert into an electron-positron pair in (i) free space (ii) an aluminum plate (iii) a lead plate? In which medium is the mean free path for conversion shorter?

9. The decay of a ρ^o meson (spin-parity = 1^-)

$$\rho^o \rightarrow \pi^+\pi^- \text{ is allowed, but}$$

$$\rho^o \rightarrow \pi^o\pi^o \text{ is not}$$

Why?

10. Can a Σ^+ hyperon (spin-parity $\frac{1}{2}^+$) have a static electric quadrupole moment? Why.

11. If a medium has a non-linear characteristic such that the velocity of propagation of a pulse increases with the amplitude of the pulse, what will happen to a disturbance for which the amplitude as a function of position is bell shaped initially? Draw a picture.

12. Energy is delivered to a clear 150 watt incandescent bulb. Immediately, about a foot away, your hand feels the heat. Now quickly touch the lamp. It still feels cold. Explain this. Explain how the hand is heated. Why is the bulb gas filled and not highly evacuated?

13. X-rays exhibit all the phenomena associated with electromagnetic waves-reflection, refraction, polarization, diffraction, interference. How would you undertake to show these phenomena with x-rays?

14. Snap your finger. Notice that one finger quickly moves from being against your thumb to contact with the palm. Keeping the other fingers extended (and not curled against the palm), snap your finger again. Explain the difference in the sound in the two cases.

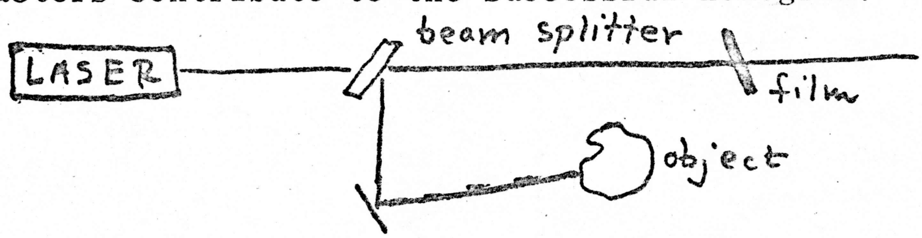
15. a) Define the term isotope

b) In the past few decades, an enormous amount of research on methods of separating isotopes has been done. Explain briefly why it is difficult to separate isotopes.

c) Name one technique used in practice for separating isotopes.

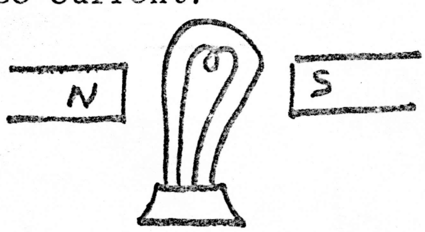
d) Why is there such intense interest in the development of efficient methods for isotope separation?

16. In holography making, the object to film distance is normally a meter or less. Because phase and not intensity are being recorded, how do the important geometrical factors contribute to the successful hologram?



17. Explain what data you would need to unambiguously verify to the layman that the earth rotates.

18. Two identical clear-glass, incandescent, carbon-filament lamps are energized, one on AC current and the other on DC current.



If now a strong horseshoe magnet is brought around each lamp, what happens to the filament in each case?

PHYSICS QUALIFYING EXAM UCI September 24, 1974

THERMODYNAMICS AND STATISTICAL MECHANICS

(Do 4 of 6 problems) Two Hours

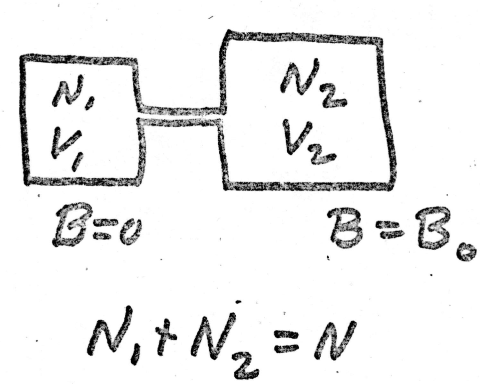
1. A gas of N particles is contained in a cylinder of cross sectional area A, but of infinite height. The gravitational potential can be taken as mgz for all z. Each molecule is rigid (i.e., it cannot vibrate) and may rotate. Each molecule has a nondegenerate electronic ground state and a three-fold degenerate first excited state at energy ϵ_o above the ground state. (All other electronic states can be neglected.) What is the entropy of the system assuming that kT is large compared to the splitting of the translational and rotational states?

2. Consider a gas of N indistinguishable, non-interacting, diatomic molecules.

   a) Calculate the total partition function including translation, vibration, and rotation. Assume kT large compared to energy spacing of rotational and translational states, but small compared to energy spacing of vibrational states.

   b) Calculate mean energy $\bar{E}$, mean pressure $\bar{p}$, and molar specific heat at constant volume.

3. Two boxes both at temperature T containing molecules of spin $\frac{1}{2}$ and magnetic moment μ are joined by a small tube. The respective volumes of the boxes are V_1 and V_2. The molecules are free to move from one box to the other. The total number of molecules is N. There is a constant magnetic field of strength B_o in box 2 and no magnetic field in box 1.



3. (Continued)

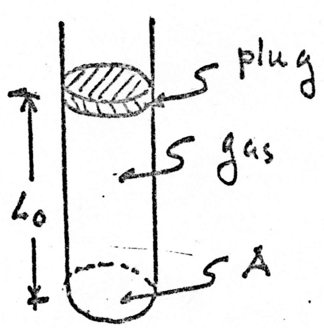
a) Calculate the partition functions for box 1 and box 2 assuming N_1 molecules in box 1 and N_2 molecules in box 2.

b) State the criterion for equilibrium between the two boxes.

c) Calculate the ratio of the number of molecules in the two boxes in equilibrium at temperature T.

4. A hot star can lose light particles easier than heavy ones. Make this argument quantitative and apply it to light electrons vs. heavy protons. How long would the solar "wind" be composed solely of electrons, and what mechanism would make the wind evolve in composition toward the charge-neutral one we have today?

5. A plug of mass M rests on top of a column of ideal gas with cross sectional area A and length L_o. The ratio of specific heats of the gas is $C_P/C_V = \gamma$.



What is the period of small adiabatic oscillations of the plug?

6. Derive the equation

a) $$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

b) Prove that C_V of an ideal gas is a function of T only.

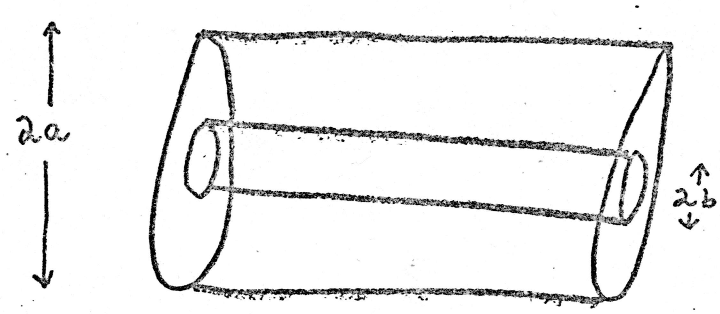
ELECTRICITY AND MAGNETISM

(Do 4 of the 6 problems) Two Hours

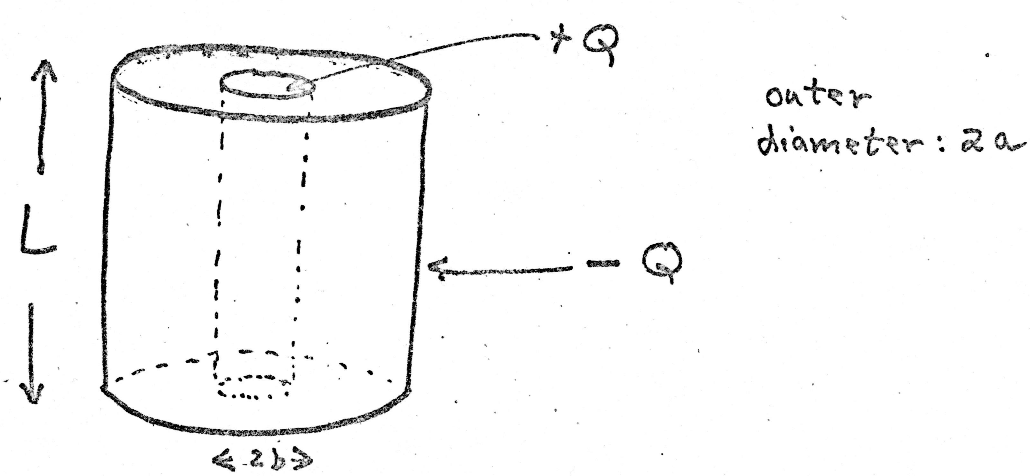
1. A square of copper is released near the earth's surface. A constant magnetic field B is parallel to the earth's surface and normal to the plane of the square at all times. The square has side length L, constant resistance R and mass m. Gravitational acceleration g is constant. Find the square's vertical velocity v as a function of time.

2. Plasma effects occur both in conventional plasmas and solids. Consider propagation of sound in a metal of n atoms per unit volume, assuming the conduction electrons are free and <u>degenerate</u> with Fermi temperature T_F. Find the velocity of sound in this metal.

3. A coaxial cavity with perfectly conducting walls and end plates is excited in its lowest mode. In terms of W, the energy stored in the electromagnetic field, determine the time-averaged pressure exerted by the field on the <u>side</u> walls and <u>end</u> walls.

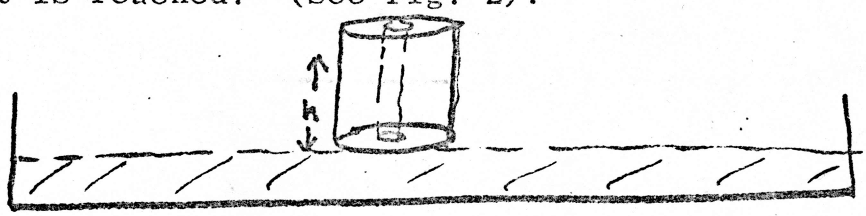


4. Consider a cylindrical capacitor which has been charged and then disconnnected from the voltage source. (see Fig. 1).



4. (Continued)

This capacitor is put into a large dish containing a liquid of dielectric constant K. The liquid rises up into the space between the plates of the capacitor until an equilibrium height is reached. (see Fig. 2).



(a) Derive an equation which could be solved to find the equilibrium height (h) of the liquid, Neglect capilarry action. From the solution of this equation it can be shown that the increase in gravitational potential energy at equilibrium is less than the decrease in electrical potential. Explain without calculation the physical origin of this difference.

(b) If the voltage source remains connected when the capacitor is put into the dish give the physical basis, without calculation, for the changes from part (a) required in the analysis.

5. An anisotropic, non-magnetic medium is placed in a large electrostatic field E_o acting in the z-direction. This makes the dielectric tensor anisotropic:

5. (Continued)

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{pmatrix}, \quad \epsilon_1 \neq \epsilon_2$$

a) Obtain the general real solution for a wave propagating through this, now anisotropic, medium in the x-direction.

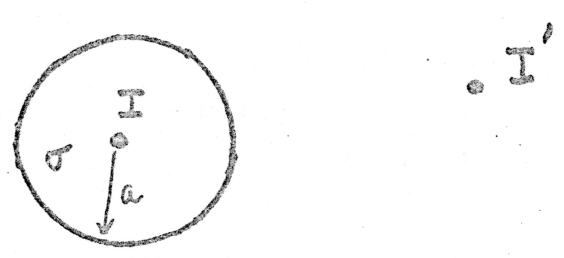
b) A linearly polarized wave of frequency ω and with polarization vector $\xi = (0, 1/\sqrt{2}, 1/\sqrt{2})$ is incident at $x = 0$. Find an expression for the distance x at which the polarization vector will be at right angles to its original direction.

6. Consider this situation, with long (effectively infinite) currents I and I′ interacting with a cylindrical conductor of radius a and conductivity σ. I and I′ are parallel. I is somehow fixed at the center of the conductor and doesn't move. I′ can move, and is a distance r from I ($r > a$).

a) Find the force F on I′ as a function of r. Consider σ infinite. Explore F(r) as a function I/I′.

b) Find the energy of the system as a function of r, with σ infinite.

c) If the answer to (b) can be infinite, for some r, what physical fact removes the effect? Estimate the maximum energy.



MATHEMATICAL PHYSICS

(Do 4 of these 5 problems) Two Hours

1. Problem:

Evaluate $$\int_o^\infty \frac{\sqrt{x}}{1+x^2}\, dx \quad .$$

Show your path of integration in the complex plane.

2. Solve the equation

$$\frac{dy}{dx} + y = xe^{-ax} + c$$

where c is a constant and y(o) = 1.

3. Consider a perfectly flexible homogeneous string of linear mass density ρ and length L, rotating with constant angular velocity Ω around a vertical axis through one end of the strong.

a) Neglecting gravity, use Hamilton's principle to show that the equation describing small transverse vibrations of the string (in a plane through the axis of rotation, rotating together with the string) is

$$\frac{\partial}{\partial x}\left((L^2 - x^2)\frac{\partial u}{\partial x}\right) - \frac{2}{\Omega^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

where u(x,t) denotes the deviation from the unperturbed position at distance x from the axis of rotation and time t. Show that the appropriate boundary conditions are

$$\begin{aligned} u(o,t) &= 0 \\ u(L,t) &= \text{finite} \quad . \end{aligned} \qquad (2)$$

b) Use the method of separation of variables to find the normal modes of the string.

c) Find the solution corresponding to the following initial conditions

$$\begin{aligned} u(x,0) &= \alpha x^3 \qquad 0 \le x \le L \\ u_t(x,0) &= 0 . \end{aligned}$$

4. The Bernoulli Numbers are defined by

$$\frac{x}{e^x - 1} = \frac{B_0}{0!} + \frac{B_1}{1!}x + \frac{B_2}{2!}x^2 + \cdots$$

Find

(a) A general relation between the B_n which allows you to evaluate them.

(b) Display B_0, B_1, B_2, B_3.

5. An infinite solid has temperature distribution $T(x,t)$. Nothing depends on y or z. Find $T(x,t)$ if $T(x,0) = \delta(x)$.

Physics Qualifying Exam UCI May 19, 1975

CLASSICAL MECHANICS

(Do All Problems)

1. A flatbed railroad car is rolling on a horizontal section of track. Assume there is no friction and therefore the speed is constant. Now consider what happens if a torrential rain storm occurs. Water falls on the railroad car and drips off onto the ground. Does this affect the speed of rolling of the car? Why? How does the speed depend on time? Assume there is no wind driving the rain, and that the vertical velocity of the rain drops is much greater than the horizontal velocity of the railroad car.

2. Consider a cube of homogeneous material with total mass M and linear dimension L. What is its moment of inertia for rotation about the body diagonal?

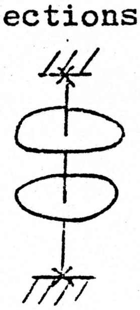
3. A piece of piano wire is stretched between two fixed supports. Two disks, each with the moment of inertia, I, are clamped onto the wire so that they divide it into three equal parts. This whole system can undergo torsional oscillation. Use the letter κ to denote the torsional Hookes Law constant for each subsection of the wire.

   a. Define a system of generalized coordinates and find the Lagrangian for the system.

   b. Find the equations of motion.

   c. Find the normal modes. Describe them, either with a sketch or in words.

   d. Discuss what would happen to the normal modes if the central section of wire is made much longer or much thinner than the end sections.



e. Discuss qualitatively what other normal modes exist for this system (i.e. other than the torsional ones).

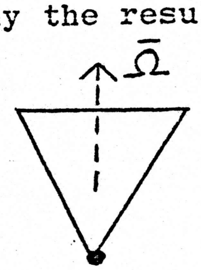
4. A simple pendulum is made by attaching a bead with mass m to two equal length massless strings and fixing the other ends of the two strings to the ends of a supporting horizontal rod. In this way the pendulum is constrained to swing only in a plane which is perpendicular to the supporting rod. The rod is rotated about its center with angular frequency Ω.

a. Express the Lagrangian of the bead in terms of a suitable generalized coordinate and find the equation of motion.

b. Find the equilibrium position of the mass as a function of Ω. Exercise some special care in discovering all physically meaningful solutions.

c. Find the frequency of small oscillations about the equilibrium as a function of Ω.

d. Sketch graphically the results of parts b and c.



5. Antiprotons can be produced by the reaction

$$p + p \rightarrow p + p + p + \bar{p}$$

Usually this is done by directing a beam of energetic protons onto a hydrogen target. In this situation what is the minimum kinetic energy of the beam protons for which this reaction can occur? When producing antiprotons with a beam having this threshold energy, what is the momentum of the antiprotons? And of one of the protons?

(Please express your answer in units in which the velocity of light c and the mass of the proton m_p both equal one.)

6. The oil-energy crisis has made popular discussions of many alternatives to current transportation systems. One such alternative is powering automobiles with energy stored in a flywheel. One of many difficulties with flywheels is that they act as gyroscopes and are difficult to turn when the auto turns or goes from level road onto a grade. How important is this problem? (Assume 300 KwHr or 10^9 joule of energy must be stored and the maximum angular frequency may reach 10^9 radians/sec).

Would such an auto have problems when parked because of the rotation of the earth? It has been suggested that by having two flywheels rotating in opposite directions one could cancel out the angular momentum and eliminate any problem. Would this work? What new problem might be introduced by such a tactic?

Physics Qualifying Exam UCI May 19, 1975

THERMODYNAMICS AND STATISTICAL MECHANICS

No textbooks or notes. Please do all problems.

1. An ideal gas has an isothermal change of volume from V_1 to V_2 at temperature T. Derive a formula for the heat absorbed.

2. A mass m of a liquid is at temperature T_1. It is mixed with an equal mass of the same liquid at temperature T_2. The system is thermally insulated, and the specific heat per gram is C_p. Calculate the change in entropy.

3. Derive a formula for the entropy of Black Body radiation in a volume V at temperature T in terms of V, T, and the fundamental constants h, c and k. Given that the energy density in a range of frequency $d\nu$ is

$$\frac{8\pi h\nu^3 d\nu}{c^3\left[e^{h\nu/kT}-1\right]}.$$

4. The Fermi-Dirac distribution law for the probability f_k that a state with energy ϵ_k is occupied is

$$f_k = \frac{1}{e^{(\epsilon_k-\mu)/kT}+1}$$

Suppose we have a Fermi gas composed of a very large number N of spin $\frac{1}{2}$ particles. Interactions between particles can be neglected and the largest kinetic energy at T=o is ϵ_o. Compute the average kinetic energy at T=o, in terms of ϵ_o, assuming the system is homogeneous, isotropic and described by non relativistic mechanics.

5. A simple pendulum of mass m hangs by a string of length ℓ. The string can be considered weightless. The pendulum hangs in a room having a gas at atmospheric pressure, and absolute temperature T. Let g be the acceleration due to gravity. Assume the pendulum is in thermal equilibrium as a result of bombardment by molecules of the gas, and its position relative to the vertical fluctuates. Compute the mean square value of the angle φ between the string and the vertical.

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Possibly Useful Information

$$\int_0^\infty e^{-a^2x^2}\,dx = \frac{1}{2a}\sqrt{\pi}$$

$$\int_0^\infty x^{n-1}e^{-x}\,dx = \Gamma(n)$$

$$\int_0^\infty \frac{\sin^2 x\,dx}{x^2} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{x^n\,dx}{e^x-1} = \begin{array}{l} 1.645\ (n=1) \\ 2.405\ (n=2) \\ 6.49\ (n=3) = \pi^4/15 \\ 24.9\ (n=4) \end{array}$$

MATHEMATICAL PHYSICS

Do 4 problems. Use <u>one</u> open book.

1. A right-circular cylinder has fixed volume, radius r and height z. Find the relation between z and r which gives minimum total surface area for fixed volume.

2. A square stretched membrane ($0 \le x \le L$, $0 \le y \le L$) is subjected to a force f(t) at the point $x = x_o$, $y = y_o$. There is a small damping proportional to the velocity of the membrane displacement. Find the general form for the membrane displacement $\Psi(x,y,t)$. Express the result in terms of the Fourier transform of f(t). Then, briefly consider the solution as the damping is removed.

3. (a) Using the identity

$$J_o(bt) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-ibt\sin x}dx$$

evaluate the integral

$$\int_o^\infty e^{-at}J_o(bt)dt$$

(b) The integral

$$I = \int_o^\infty dx\, J_n^{\,2}(dx)\left[\frac{d}{dx}\left(axe^{-a^2x^2}\right)\right]$$

can in general be either positive or negative, depending on the constants d and a. Without doing the integral, find a qualitative condition between d and a which will insure that I is positive.

4. Consider the matrix equation

$$(L_o + Q)u = \lambda u$$

where

$$L_o = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, Q = \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and $\epsilon << 1$, i.e., Q is a perturbation.

(a) Find the zero-order eigenvalues, λ_o, and the zero-order column vectors u_o.

(b) Find the perturbed eigenvalues λ_1.

5. Calculate the principal value of

$$\int_o^\infty \frac{\ln x}{x^{1/2}(1-x)}\, dx$$

Physics Qualifying Exam UCI May 20, 1975

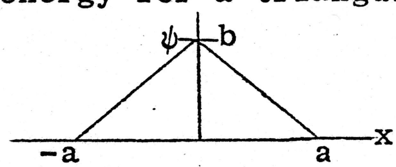
QUANTUM MECHANICS I

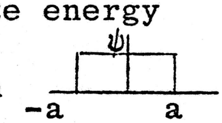
You may use one Quantum Mechanics textbook.

1) Consider a 1 dimensional harmonic oscillator

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{k}{2}x^2$$

Minimize the energy for a triangular trial wave function



Compare your calculated value to the true ground state energy for H. Discuss the use of a square trial wave function 

.

2) Consider a particle of mass m scattered by a square well potential with a hard core

$$V(r) = +\infty \qquad r < c$$

$$= -V_o \qquad c < r < a$$

$$= 0 \qquad r > a$$

a) Calculate the S wave phase shift δ as a function of energy.

b) For the case $c = o$ calculate δ in the Born approximation. Compare with your exact result from a) at high energy.

3) A deuteron (hyrdogen 2 nucleus) consists of a neutron and proton in a (total) angular momentum state $J = 1$.

a) Using the Clebsch-Gordon Coef in the table calculate all possible wave functions $\psi_{L,S}^{J=1,J_z=1}$ in terms of the intrinsic spin (S) wave functions χ for the n and p and an orbital angular momentum (L) wave function ϕ. (Do this neatly.)

b) Using

$$\mu_D = \langle \psi^{J=1, J_z=1} \; \mu_z \; \psi^{J=1, J_z=1} \rangle$$

$$\mu_z = (\underline{\mu})_z = (2\mu_n \underline{S}_n + 2\mu_p \underline{S}_p + \tfrac{1}{2} \underline{L})_z$$

with $\mu_n = -1.91$ (in units of nuclear Bohr magnetons) and $\mu_p = 2.79$.

Calculate various possible magnetic moments μ_D of deuteron using the wave functions from part a).

c) Compare result with the experimental value μ_D of .86. Conclude that the deuteron is predominately an $S = 1$, $L = 0$ state. What other state can mix (via strong interactions) with it? What percentage mixture of this state would give the experimental μ_D? How would you observe the presence of the other state experimentally?

TABLE VII

CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

1/2X1/2

m_1 m_2 \ J M	1 +1	1 0	0 0	1 -1
+1/2 +1/2	1			
+1/2 -1/2		$\sqrt{1/2}$	$\sqrt{1/2}$	
-1/2 +1/2		$\sqrt{1/2}$	$-\sqrt{1/2}$	
-1/2 -1/2				1

1X1/2

m_1 m_2 \ J M	3/2 +3/2	3/2 +1/2	1/2 +1/2	3/2 -1/2	1/2 -1/2	3/2 -3/2
+1 +1/2	1					
+1 -1/2		$\sqrt{1/3}$	$\sqrt{2/3}$			
0 +1/2		$\sqrt{2/3}$	$-\sqrt{1/3}$			
0 -1/2				$\sqrt{2/3}$	$\sqrt{1/3}$	
-1 +1/2				$\sqrt{1/3}$	$-\sqrt{2/3}$	
-1 -1/2						1

3/2X1/2

m_1 m_2 \ J M	2 +2	2 +1	1 +1	2 0	1 0	2 -1	1 -1	2 -2
+3/2 +1/2	1							
+3/2 -1/2		$\sqrt{1/4}$	$\sqrt{3/4}$					
+1/2 +1/2		$\sqrt{3/4}$	$-\sqrt{1/4}$					
+1/2 -1/2				$\sqrt{1/2}$	$\sqrt{1/2}$			
-1/2 +1/2				$\sqrt{1/2}$	$-\sqrt{1/2}$			
-1/2 -1/2						$\sqrt{3/4}$	$\sqrt{1/4}$	
-3/2 +1/2						$\sqrt{1/4}$	$-\sqrt{3/4}$	
-3/2 -1/2								1

2X1/2

m_1 m_2 \ J M	5/2 +5/2	5/2 3/2	3/2 +3/2	5/2 +1/2	3/2 +1/2	5/2 -1/2	3/2 -1/2	5/2 -3/2	3/2 -3/2	5/2 -5/2
+2 1/2	1									
+2 -1/2		$\sqrt{1/5}$	$\sqrt{4/5}$							
+1 +1/2		$\sqrt{4/5}$	$-\sqrt{1/5}$							
+1 -1/2				$\sqrt{2/5}$	$\sqrt{3/5}$					
0 +1/2				$\sqrt{3/5}$	$-\sqrt{2/5}$					
0 -1/2						$\sqrt{3/5}$	$\sqrt{2/5}$			
-1 +1/2						$\sqrt{2/5}$	$-\sqrt{3/5}$			
-1 -1/2								$\sqrt{4/5}$	$\sqrt{1/5}$	
-2 +1/2								$\sqrt{1/5}$	$-\sqrt{4/5}$	
-2 -1/2										1

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta\,; \quad Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right); \quad Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_3^0 = \sqrt{\frac{7}{4\pi}}\left(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta\right); \quad Y_3^1 = -\frac{1}{4}\sqrt{\frac{21}{4\pi}}\sin\theta\left(5\cos^2\theta - 1\right)e^{i\phi}$$

$$Y_3^2 = \frac{1}{4}\sqrt{\frac{105}{2\pi}}\sin^2\theta\cos\theta\, e^{2i\phi}\,; \quad Y_3^3 = -\frac{1}{4}\sqrt{\frac{35}{4\pi}}\sin^3\theta\, e^{3i\phi}$$

$$\left(Y_l^m\right)^* = \left(-1\right)^m \; Y_l^{-m}$$

1X1

m_1 m_2 \ J M	2 +2	2 +1	1 +1	2 0	1 0	0 0	2 -1	1 -1	2 -2
+1 +1	1								
+1 0		$\sqrt{1/2}$	$\sqrt{1/2}$						
0 +1		$\sqrt{1/2}$	$-\sqrt{1/2}$						
+1 -1				$\sqrt{1/6}$	$\sqrt{1/2}$	$\sqrt{1/3}$			
0 0				$\sqrt{2/3}$	0	$-\sqrt{1/3}$			
-1 +1				$\sqrt{1/6}$	$-\sqrt{1/2}$	$\sqrt{1/3}$			
0 -1							$\sqrt{1/2}$	$\sqrt{1/2}$	
-1 0							$\sqrt{1/2}$	$-\sqrt{1/2}$	
-1 -1									1

3/2X1

m_1 m_2 \ J M	5/2 +5/2	5/2 +3/2	3/2 +3/2	5/2 +1/2	3/2 +1/2	1/2 +1/2	5/2 -1/2	3/2 -1/2	1/2 -1/2	5/2 -3/2	3/2 -3/2	5/2 -5/2
+3/2 +1	1											
+3/2 0		$\sqrt{2/5}$	$\sqrt{3/5}$									
+1/2 +1		$\sqrt{3/5}$	$-\sqrt{2/5}$									
+3/2 -1				$\sqrt{1/10}$	$\sqrt{2/5}$	$\sqrt{1/2}$						
+1/2 0				$\sqrt{3/5}$	$\sqrt{1/15}$	$-\sqrt{1/3}$						
-1/2 +1				$\sqrt{3/10}$	$-\sqrt{8/15}$	$\sqrt{1/6}$						
+1/2 -1							$\sqrt{3/10}$	$\sqrt{8/15}$	$\sqrt{1/6}$			
-1/2 0							$\sqrt{3/5}$	$-\sqrt{1/15}$	$-\sqrt{1/3}$			
-3/2 +1							$\sqrt{1/10}$	$-\sqrt{2/5}$	$\sqrt{1/2}$			
-1/2 -1										$\sqrt{3/5}$	$\sqrt{2/5}$	
-3/2 0										$\sqrt{2/5}$	$-\sqrt{3/5}$	
-3/2 -1												1

2X1

m_1 m_2 \ J M	3 +3	3 +2	2 +2	3 +1	2 +1	1 +1	3 0	2 0	1 0	3 -1	2 -1	1 -1	3 -2	2 -2	3 -3
+2 +1	1														
+2 0		$\sqrt{1/3}$	$\sqrt{2/3}$												
+1 +1		$\sqrt{2/3}$	$-\sqrt{1/3}$												
+2 -1				$\sqrt{1/15}$	$\sqrt{1/3}$	$\sqrt{3/5}$									
+1 0				$\sqrt{8/15}$	$\sqrt{1/6}$	$-\sqrt{3/10}$									
0 +1				$\sqrt{6/15}$	$-\sqrt{1/2}$	$\sqrt{1/10}$									
+1 -1							$\sqrt{1/5}$	$\sqrt{1/2}$	$\sqrt{3/10}$						
0 0							$\sqrt{3/5}$	0	$-\sqrt{2/5}$						
-1 +1							$\sqrt{1/5}$	$-\sqrt{1/2}$	$\sqrt{3/10}$						
0 -1										$\sqrt{6/15}$	$\sqrt{1/2}$	$\sqrt{1/10}$			
-1 0										$\sqrt{8/15}$	$-\sqrt{1/6}$	$-\sqrt{3/10}$			
-2 +1										$\sqrt{1/15}$	$-\sqrt{1/3}$	$\sqrt{3/5}$			
-1 -1													$\sqrt{2/3}$	$\sqrt{1/3}$	
-2 0													$\sqrt{1/3}$	$-\sqrt{2/3}$	
-2 -1															1

Note: When calculating terms which are linear in the above coefficients (e.g., interference, polarization), the sign convention becomes important. This table follows the one in Blatt and Weisskopf, Edmonds, Rose, Condon and Shortley, etc. Other authors (e.g., Schiff, Bethe and de Hoffmann) use different conventions.

Physics Qualifying Exam UCI May 21, 1975

QUANTUM MECHANICS II

Do All Problems

1. Using the uncertainty relation calculate the maximum time that a ball bearing (mass = 1 gm) is likely to remain on the top of a perfectly flat table with radius R = 1 meter. (Do not consider such real effects as building vibrations and continental drift. Concentrate on the quantum mechanical upper limit.)

2. Consider the helium atom. Estimate the ground state energy either by using perturbation theory or by the variational method.

Potentially useful integrals and formulas.

$$\int d^3r\, e^{-2kr} = \frac{\pi}{k^3}$$

$$\int d^3r\, \frac{e^{-2kr}}{r} = \frac{\pi}{k^2}$$

$$\int d^3r_1 d^3r_2 \frac{e^{-2k(r_1 + r_2)}}{|\vec{r}_1 - \vec{r}_2|} = \frac{5}{8}\frac{\pi^2}{k^5} ,$$

$$\nabla_r^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

3. a. In atomic physics, the spin orbit interaction has the form $A\,\vec{\ell} \cdot \vec{s}$. Derive an expression for the coefficient A in a hydrogenic atom. Don't worry about factors of two, feel free to use qualitative arguments, but get the dependence on the relevant variables correct. Estimate the magnitude of the fine structure splitting in the optical spectrum of sodium.

b. An electron in a state of orbital angular momentum ℓ is placed in a weak magnetic field. Find an expression for the g-factor.

QUALITATIVE PHYSICS

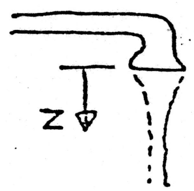
Do all problems.

1. The human body can expel air with a maximum expiratory pressure of about 120 mm Hg. The Indians of the Amazon make use of this pressure to accelerate a poisoned dart in a blowgun.
   a. Neglecting friction, calculate the muzzle velocity v_m of a 1 gram dart in a 3 meter blowgun of inner cross-sectional area of 1 cm^2.
   b. A resistive force due to the compression of the air ahead of the dart is

$$F_{res} \approx \rho_{air} A v^2$$

   where $\rho_{air} = 1.3\ kg/m^3$. Assume the dart attains a constant velocity state inside the muzzle. How long would you make a blowgun to guarantee that the exit velocity of the dart is within 10% of v_m?

2. A continuous stream of water effluxing downward from the nozzle of a faucet has a characteristic profile.



   a. Assuming that each element of the fluid is in free fall, find the velocity of the fluid a distance z below the nozzle.
   b. What is the profile of the water stream as a function of z? (The profile corresponds to cross-sectional area.)

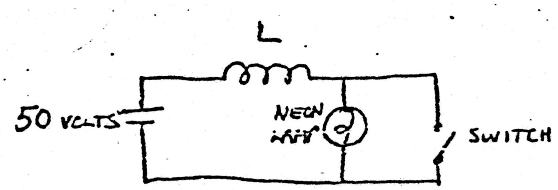
3. POTPOURRI:

a. Air exhaled from a wide open mouth feels warmer to the back of the hand than air exhaled from a nearly closed mouth even though the volume of air per second emitted is the same. Explain.

b. From an atomic viewpoint, why does a blackbody radiate more heat energy than a non-black body which is identical in construction except for color, assuming both are initially at the same temperature?

c. A stiff card is placed on top of a drinking glass. A coin is put on the card above the center of the glass and a snap of the first finger off the thumb delivers an appropriate impulsive blow to the edge of the card. What is the condition for the coin to remain on the card?

4. Given the choke circuit with a battery, an inductor, a neon lamp and a switch. Close the switch, then open it. If the neon lamp will not glow below an applied voltage of 90 volts, will the lamp glow at any time during or after the closing or opening of the switch? Explain the physics.



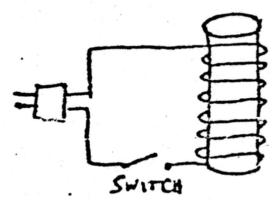
5. An iron bar is suspended from a spring above an electromagnet, their axes coinciding. When a current is passed through the electromagnet, the iron bar is drawn into the core of the electromagnet. (There initially being only an open, non-filled core.)

a. Give an explanation for the motion of the iron bar.

b. Qualitatively explain the subsequent motion of the bar.

c. Does Lenz's law apply here? Explain.

6. A repulsion coil constructed from 1000 turns of #12 insulated copper wire around a tube (2 cm diameter) is plugged into a 120 volt outlet. The 10 amp fuse blows up.
   a. By substituting a 10 amp slow blow fuse, the operation of the coil proceeds uninterrupted. Why?
   b. Add iron rods (these rods are long enough to protrude out the top) down the core center. Will the current drawn at 120 volts increase or decrease? Explain.
   c. An aluminum ring placed around the coil just above the center jumps upward when the coil is being energized. Explain the physics.
   d. A nonconducting ring does not jump during energization. Will it have an EMF around it when placed in the changing field?



7. An optical diffraction grating is subject to distortions resulting from the lack of care and quality control. Qualitatively, in terms of the resolution, linewidth and angular dispersion, explain the features of the diffraction pattern introduced by
   a. a non-planar grating
   b. unequal slit spacings
   c. unequal slit widths
   d. tilt with respect to the incident beam
   e. non-uniform slit widths (trapezoiding).

8. Estimate time in seconds for:
   a. age of universe
   b. lifetime of excited electron in an atom
   c. collision time of an air molecule
   d. time of flight of light from sun to earth
   e. decay time of LCR (inductance, capacitance, resistance) circuit with
      $L = 1$ Henry , $C = 1\ \mu F$, $R = 20\Omega$
   f. average half-life of C^{14} for radiocarbon dating
   g. sound to travel down a 100 meter well and echo back.

9. A recent safety rope invention in mountaineering is an elastic rope which stretches by an amount ΔL proportional to the applied tension T:

$$T = \gamma \frac{\Delta L}{L_0}$$

L_0 being the length at zero tension. Assume that a climber falls from a height L_0 above the last piton where the rope is anchored.
   a. Find the velocity v_0 after he falls a distance $2L_0$ and choose this point as the zero of time and where $z = 0$.
   b. Qualitatively graph the motion of the falling body.
   c. Given $\gamma = 10^4$ lbs. and a 150 lb. climber, what is the maximum tension T and show that this maximum T is <u>independent</u> of the height of the fall, $2L_0$.

10. a. Two church bells which appear to be identical are struck by hammers imparting the same impulse. However, they emit a different quality of sound, the better one resonating for twice as long as the other. Explain the differences in their sound characteristics in physics terms such as frequency spectrum, the Q, and the decay time.
   b. Organ pipes also exhibit sound resonance. Independent of the excitation method, organ pipes produce harmonics. Using Fourier analysis, qualitatively discuss the harmonics expected from open and closed (at one end) pipes.

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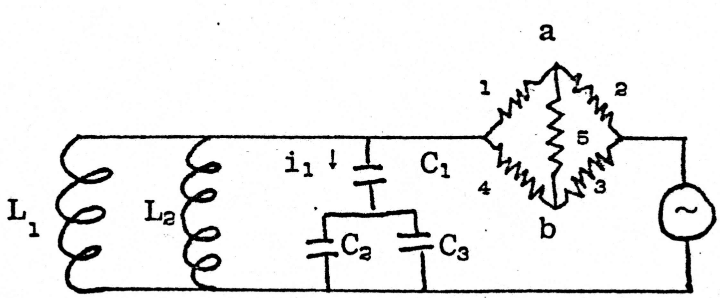
ELECTRICITY AND MAGNETISM

Do 4 Problems (Closed Book)

1. A parallel plate capacitor of area A, plate separation ℓ, and dielectric ε is charged and then disconnnected from the charging source.
   a. Calculate $\vec{E}$, $\vec{D}$, the potential difference between the plates, and the capacitance.
   b. What force is required to keep the plates apart?
   c. How much work is required to remove the dielectric?, to completely disassemble the capacitor?
   d. Under what condition is the fringing field negligible?

2.



$V = V_o \sin \omega t$

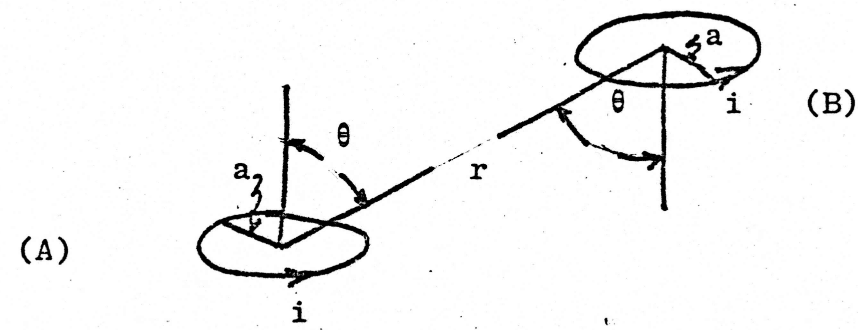
$V_a = V_b$

Write differential equation for the currents i, in the above circuit. Solve for steady steady condition.

3. Calculate:

a. the force between the two current carrying loops shown in the figure.

b. the torque exerted by one loop on the other.



$\frac{a}{r} << 1$

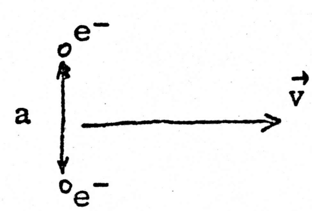
4. Write the equation for the propagation of a plane electromagnetic wave in a conducting medium.

a. Assuming a monochromatic wave, how far will it penetrate into the medium?

b. Is the field transverse or longitudinal? Discuss your answer.

5. a. What is the force per unit length between two very long parallel current-carrying conductors? Apply this result to explain the self-compression of a current-carrying neutral plasma (pinch effect).

b. Consider two electrons moving parallel to each other at a velocity v in the laboratory. What is the force between the electrons for $v = 0$, $v = c$?



Physics Qualifying Exam UCI September 22, 1975

CLASSICAL MECHANICS

Text: Goldstein or Landau.

Everyone do the first problem, then any 3 others.

1. A uniform universe of matter, initially homogeneous with density ρ_o, pressure P_o, fluid velocity $V_o = 0$, is subject to self-gravitation, described by

$$\vec{\nabla} \times \vec{g} = 0$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho$$

where $\vec{g}$ is the gravitational field. The mass fluid obeys

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{v}) = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \vec{g}$$

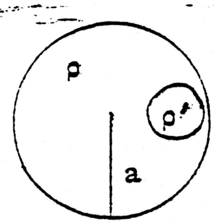
and $C_s^2 \equiv$ sound speed $\equiv \frac{\partial p}{\partial\rho}$

a. Find the "dispersion relation" for this initially homogeneous universe.

b. Some density perturbations of wave length $\frac{2\pi}{k}$ are stable, some not. Which region of k-space is stable? What is the critical k required for instability?

c. Suppose the mass fluid has some rotation at angular frequency Ω = constant. How would you expect stability to be affected? Guess the growth rate.

2. Take the earth to be a sphere of density ρ, radius a. Suppose there is a hollow sphere of radius R located somewhere inside the earth, filled with matter of density ρ'. Find



a. the acceleration $\vec{g}$ due to gravity at the point on earth's surface nearest the hollow sphere.

b. $\vec{g}$ at the point on the surface farthest from the hollow sphere.

3. Consider a particle of mass m in the laboratory system moving parallel to the x-axis and at a height Z_o above it for large negative x, where its velocity is (v_o,o,o). It interacts with a potential $U(\underset{\sim}{x})$ centered at the origin of coordinates. Assuming that $\underset{\sim}{F}(\underset{\sim}{x}) = -\nabla U(\underset{\sim}{x})$ is weak, solve the equations of motion of the particle

$$m\ddot{\underset{\sim}{x}} = \underset{\sim}{F}(\underset{\sim}{x})$$

to first order in $\underset{\sim}{F}(x)$ to obtain $m\dot{x}_1(t)$ $m\dot{x}_2(t)$, and $m\dot{x}_3(t)$. Specialize to the case of the Kepler problem for which $U(\underset{\sim}{x}) = \frac{\alpha}{|\underset{\sim}{x}|}$ and obtain the angle of deflection of the particle

$$\theta_1 \approx \frac{m\dot{x}_3(\infty)}{m\dot{x}_1(\infty)} = \frac{2\alpha}{mv_o z_o^2} .$$

Note: $\vec{x} = (x_1,x_2,x_3)$

4. A uniform rod slides with its ends on a smooth vertical circle of radius a. If the rod subtends an angle of $2\theta < 180°$ at the center of the circle, find the frequency of small oscillations of the rod.

5. In search of more living space, an intelligent race plans to redistribute their planet's mass into a giant ring, centered on their star and spinning with angular frequency Ω. Their leading physicist argues that this design is stable. Evaluate this claim. (Drawing pictures is usually a clearer method of explanation.)

Physical Qualifying Examination UCI September 22, 1975

THERMODYNAMICS AND STATISTICAL MECHANICS

Do four problems

1. Suppose we release a cloud of cigar smoke at Irvine. Assume the smoke particles behave like other molecules in the air. Further, take our atmosphere to be at constant temperature everywhere and let there be no winds (a stagnant atmosphere; no convection). How much time (in seconds) will pass before many smoke particles (say, 10%) have spread halfway around the earth? Make a good numerical estimate.

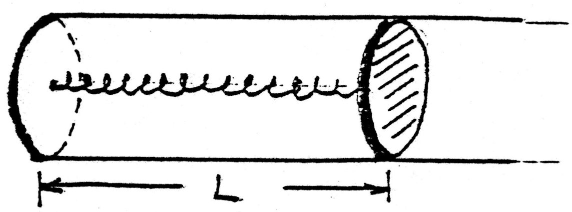
2. Consider a container of total volume V separated into two equal parts by an impermeable partition. Side one is filled with N noninteracting spin $\frac{1}{2}$ particles of mass m and side two is filled with N noninteracting spin $\frac{1}{2}$ particles of zero mass. (For instance neutrons and neutrinos respectively.) Assume that the walls are impermeable to both kinds of particles and that temperature is zero.

   a) Calculate the ratio of total energies in the two sides. (Hint: For the zero mass particles, the single particle energy $e = pc = \hbar ck$ where p = momentum and k = wave vector. For the massive particles, $\epsilon = \hbar^2 k^2/2m$. Use Fermi-Dirac statistics on these degenerate ideal gasses.)

   b) If the partition is now released, and becomes free to move (but remains impermeable to both energy and particles), calculate the ratio of equilibrum volumes.

3. Consider a metal with free electrons in it. Calculate the speed of sound in this metal. The electrons are degenerate with Fermi temperature T_F, and density n. How does this differ in basic physical ways from the sound speed in, say, a rock?

4. A gas of N noninteracting classical particles is contained inside a pipe of cross sectional area A. One end of the pipe is closed while the other is fitted with a movable piston attached by a spring, with spring constant α to the fixed end. When the piston is against the closed wall the spring is in its neutral position. The system is at temperature T. Let L be the extension of the spring.



a. What is the average value of L?

b. What is the fluctuation around this value?

5: A body obeys the equation of state

$$pV^{\alpha} = dT^{\beta} \qquad \text{(d constant)}$$

At a fixed volume V_o the specific heat is found to be independent of temperature and equal to c_v. Express the internal energy U and entropy S as a function of T and V.

Physics Qualifying Examination UCI September 23, 1975

QUANTUM MECHANICS I

1. Let E_o and E_1 be the first two energy levels of a quantum mechanical system. Let ψ_o be the wave function of the ground state. We now perform a variational calculation for the ground state and find the approximate energy level and wave function E_v and ψ_v. ψ_o and ψ_v are normalized to 1. Show that

$$\frac{E_v - E_o}{1 - |\langle \psi_o | \psi_v \rangle|^2} \geq E_1 - E_o$$

2. A spin $\frac{1}{2}$ particle with gyromagnetic ratio g and mass m is initially polarized in the plus z direction. A magnetic field of strength B is turned on at time t = 0 in the x direction. What is the probability of finding the polarization in the plus z direction at a time t later?

3. Consider scattering in the extreme low energy limit from a very thin, attractive, spherical shell potential

$$V(r) = -V_o \text{ for } a < r < a + \Delta a$$
$$= 0 \text{ elsewhere}$$

Find the scattering amplitude and total cross section. You may use the Born approximation and the limit $ka \ll 1$; indicate the restriction on the thickness Δa for this approximation to be valid.

4. a) Consider a charged particle bound in an isotropic 3-dimensional harmonic oscillator potential. Without necessarily deriving the results, what are the energy levels and their degeneracies?

4. b) Suppose a uniform external electric field is imposed on the system in (a). How are the energy levels and degeneracies affected?

   c) Suppose a uniform external magnetic field is imposed on the system in (a). Indicate how the energy levels and degeneracies are affected to lowest non-vanishing order in the field strength.

   d) Is parity a good quantum number for the system described in (a)? (b)? (c)?

5. Consider the elastic scattering of two spinless particles in an angular momentum channel ℓ, supposing that competing inelastic reactions also occur in the same channel. The partial wave elastic scattering amplitude is written

$$f_\ell = \frac{1}{k} e^{i\delta_\ell} \sin\delta_\ell \text{ where k is the wave number}$$

a) Show that the phase shift δ_ℓ must necessarily be complex.

b) What must be the algebraic sign of the imaginary part of the phase shift?

c) Derive an expression for the elastic scattering cross section in the ℓ channel.

d) Derive an expression for the inelastic scattering cross section in the ℓ channel.

e) Derive an expression relating the total cross section in the ℓ channel and the imaginary part of the partial wave amplitude.

f) What is the maximum possible value for the ratio of the inelastic to the elastic scattering cross section?

Physics Qualifying Exam UCI September 23, 1975

MATHEMATICAL PHYSICS

Text: Arfken or equivalent.

Do 4 problems.

1. Consider a perfectly flexible homogeneous string of linear mass density ς and length L, rotating with constant angular velocity Ω around a vertical axis through one end of the string.

a. Neglecting gravity, use Hamilton's principle to show that the equation describing small transverse vibrations of the string (in a plane through the axis of rotation, rotating together with the string) is

$$\frac{\partial}{\partial x}\left((L^2 - x^2)\frac{\partial u}{\partial x}\right) - \frac{2}{\Omega^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

where $u(x,t)$ denotes the deviation from the unperturbed position at distance x from the axis of rotation and time t. Show that the appropriate boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= \text{finite} \end{aligned} \qquad (2)$$

b. Use the method of separation of variables to find the normal modes of the string.

(continued)

c. Find the solution corresponding to the following initial conditions

$$u(x,o) = \alpha^{3} \cdot o \leq x \leq L \quad .$$

$$u_t(x,o) = o$$

2. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)\cosh\left(\frac{\pi x}{2}\right)}$$

3. A falling object in a resisting medium obeys the equation

$$m\,\frac{d^2x}{dt} = mg - A\,\frac{dx}{dt}$$

Given that $x(o) = o$ and $\left.\frac{dx}{dt}\right|_{t=o} = o$, find $x(t)$ and $\frac{dx}{dt}$.

4. Consider the series

$$1 - n(n+1)\,\frac{x^2}{2!} + n(n-2)(n+1)(n+3)\,\frac{x^4}{4!} - \quad .$$

For what values of x does this series converge?

5. Find to two significant figures a root of

$$f = x^3 + x - 1 = 0$$

for x between 0.5 and 1.0.

6. Consider the equation

$$\frac{dy}{dx} = e^{y/x}$$

and quickly sketch a few properties of the solution in x-y space. Then suppose $y(1) = 0$. Find a series expansion in $u = x - 1$, valid near $x = 1$. Find the series to order u^3.

Physics Qualifying Exam . Sept. 24, 1975

ELECTRICITY AND MAGNETISM

Do 4 problems
Text: Jackson

1. In general, the electrical conductivity of a pure metal is a tensor $\vec{\sigma}$ defined by

$$\vec{j} = \vec{\sigma}\,\vec{E} = ne\,\vec{v}_D$$

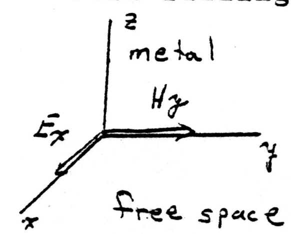
where $\vec{j}$ is the current density, $\vec{E}$ the electric field, and $\vec{v}_D$ the average drift velocity. Calculate the electrical conductivity of a free electron gas in the present of a magnetic field $B\hat{z}$ in the z direction. Assume that the effect of collisions can be described by an energy independent collision time τ. Use classical theory.

2. A square of copper is released near the earth's surface. A magnetic field

$$B = B_o \left(\frac{A}{r}\right)^3 , \quad \text{exist for } r > R$$

where R is the earth's radius, r the distance from the earth's center and A a constant. Assume the square is released at some radius R' and during its fall $\vec{B}$ is parallel to the earth's surface and normal to the plane of the square at all times. The square has side length L, thickness d, constant resistivity ρ, and mass m. Gravitational acceleration g is constant. Find the square's vertical velocity v as a function of time.

3. The surface impedance of a semiinfinite metal filling the half space $z > 0$ is defined as



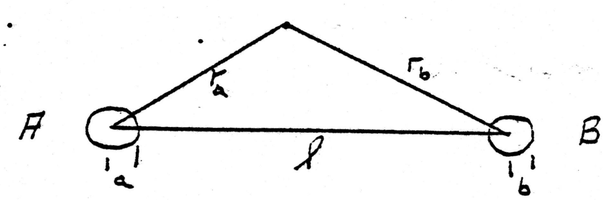
$$Z = \frac{E_x(z=0)}{\int_0^\infty J_x(z)\,dz} .$$

Show from Maxwell's equations that Z can also be written as

$$Z = \frac{E_x(z=0)}{H_y(z=0)} = i\,\mu_o\,\omega \frac{E_x(z=0)}{\left.\frac{\partial E_x}{\partial z}\right|_{z=0}}$$

in S.I. units. (Hint: assume $\vec{E} = \hat{i}\,E_x e^{i(kz-\omega t)}$; $\vec{H} = \hat{j}\,Hy\,e^{i(kz-\omega t)}$ and neglect the displacement current.)

4. Two small spherical electrodes A and B of radius a and b are located a distance ℓ apart in an infinite conducting medium of conductivity σ. Find the resistance between the two electrodes.

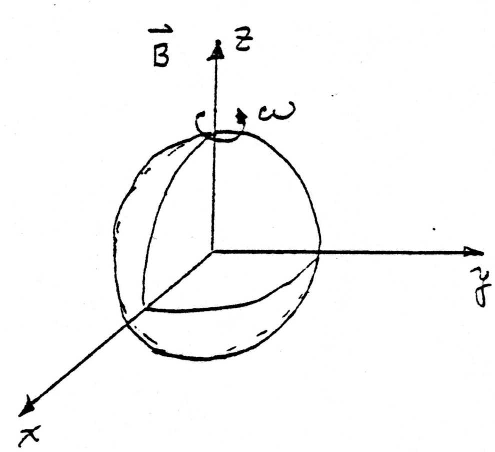


(Hint: First find the potential V at an arbitrary point r_a from A and r_b from B for a current I flowing between the two electrodes by using the principle of superposition. Then find $V_a - V_b$.)

5. Find the electric potential everywhere produced by a dielectric sphere rotating with angular frequency ω about the direction of a uniform magnetic field. Assume that the polarization $\vec{P}$ is given by

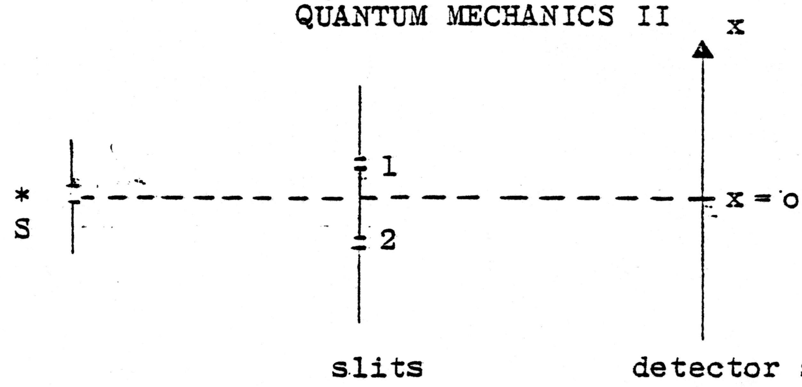
$$\vec{P} = (\epsilon - 1)\{\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\}$$

where ϵ is the dielectric constant of the sphere and v is velocity of the dielectric. Assume that the conductivity is zero and the total charge on the sphere is zero.



QUANTUM MECHANICS II

1.



Suppose a strong source, S, emits a high flux of monoenergetic electrons which pass through slits 1 and 2, and fall on a detector capable of measuring the position x of the electrons.

Sketch, as a function of x, the rate at which electrons are detected on the screen under the following circumstances

a) Slit 1 open, 2 closed

b) Slit 2 open, 1 closed

c) Slits 1 and 2 both open.

In what way would the patterns in (a), (b), (c) differ if the source were very weak, i.e., if there were a negligible probability of two or more electrons arriving simultaneously at the screen within the resolving time of the detector?

2. The Hamiltonian for a free Dirac particle is

$$H = c\,\vec{\alpha}\cdot\vec{p} + \beta m c^2$$

a) What is the velocity operator?

b) What are its eigenvalues?

c) Discuss the significance of the result (b).

3. The 21 cm line used by the radio astronomers is a magnetic dipole transition between the hyperfine levels of the ground state of atomic hydrogen.

a) What are the various quantum numbers of these energy levels?

b) Explain why the transition is magnetic dipole.

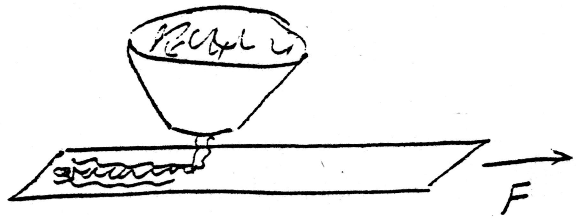
c) Discuss the effect of a uniform <u>weak</u> magnetic field on the spectral line (i.e. the weak field Zeeman effect).

Physics Qualifying Exam UCI September 24, 1975

GENERAL PHYSICS

No text. Answer 10.

1.



Sand drops from a stationary container at a rate $\frac{dm}{dt}$ (m is the mass of sand) onto a belt moving with a constant velocity v.

a. What force F is required to keep the belt moving at speed v?

b. Show that the power supplied by the force F is twice the rate of increase of kinetic energy.

c. Where has the remainder of the power gone?

2. Coherent vs Incoherent Radiating Systems

Consider n similar antennas emitting linearly polarized electromagnetic radiation of wavelength λ and velocity c.

The antennas are located along the x axis at a separation λ from each other. An observer is located on the x axis at a great distance from the antennas. When a single antenna radiates, the observer measures and intensity (mean square electric field amplitude) equal to I.

a. If all antennas are driven in phase by the same generator of frequency $\nu = c/\lambda$ what is the total intensity measured by the observer?

(Cont'd next page)

b. If the antennas all radiate at the same frequency $\nu = c/\lambda$ but with completely random phase, what is the mean intensity measured by the observer?

3. One of the many ecological problems associated with nuclear electrical power generating plants is the excess thermal energy that must be discharged into the environment (thermal pollution). Why does a nuclear powered generating station present a more serious problem than a fossil fuel powered generating station? (Hint: the steam produced by a fossil fuel heated boiler is considerably hotter than that produced by a nuclear reactor.)

4. One of the most exciting new discoveries in recent years in low temperature physics has been a phase transition in liquid He^3 at very low temperatures (< 3mk). This new phase is presumably a superfluid more like that in a superconductor than like superfluid liquid He^4. Comment on the nature of a superfluid phase in a system composed of Bose particles (He^4) and a system composed of Fermi particles (He^3 and electrons in a superconductor).

5. What is the meaning of the term "adiabatic invariant" in classical mechanics? Apply this concept to the discussion of the damping of transverse oscillations in a relativistic particle accelerator. Assume that these oscillations can be described by the motion of a particle in a parabolic potential and that the damping arises from the relativistic increase of mass taking place during the acceleration. Use English sparingly; stress mathematics.

6. When you see a piece of metal, you can generally tell that it is a metal. Discuss why you can tell a metal from other forms of solids. Are there non-metals which can look like a metal?

7. Consider the elastic scattering process

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

for an incident anti neutrino on an electron. Derive the equation relating initial and final neutrino energies and the scattering angle.

8. A. In a glass of water at 0°C, there floats a cube of ice. Enough heat is added to melt the ice at 0°C. Does the water level rise, fall or remain constant? Why? The water temperature is raised to 2°C. Does the water level rise, fall or remain constant?

9. How do you explain the fact that a small child can "pump up" a swing without direct contact with the ground?

10. During recent weeks, several miniature rockets have been launched in the campus central park. These rockets have small engines that provide typically 10 Newton-seconds of impulse. With such an engine, what is the approximate maximum altitude of a 50 gm vehicle? What additional information would you need before you could make a more accurate calculation of the maximum altitude?

11. In Physical Review Letters last month appeared an article claiming to have experimentally detected a magnetic monopole. What properties or characteristics are conventionally assumed of the monopole? What makes it easy to detect, in principle?

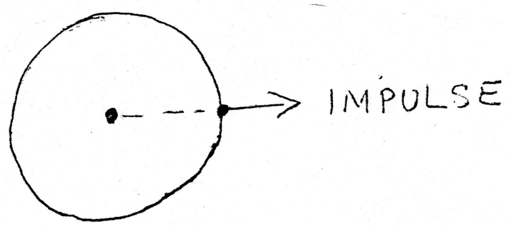
12. Semiconductor devices such as transitors, diodes, etc., fabricated on silicon surfaces, are being made smaller and smaller every day. (The smallest today are $10^{-6} \times 10^{-6}$m). What physical limitations do you expect to determine the ultimate minimum size?

13. We are surrounded with solid state devices that have come from physics laboratories. Explain briefly the physical processes involved in

    magnetic core memory of computers
    light emitting diodes (LED)
    metal-oxide-semiconductor field effect transistors (MOSFET)
    Gunn oscillator
    charge coupled memory for computers
    Superconducting Quantum Interference Devices (SQUID)

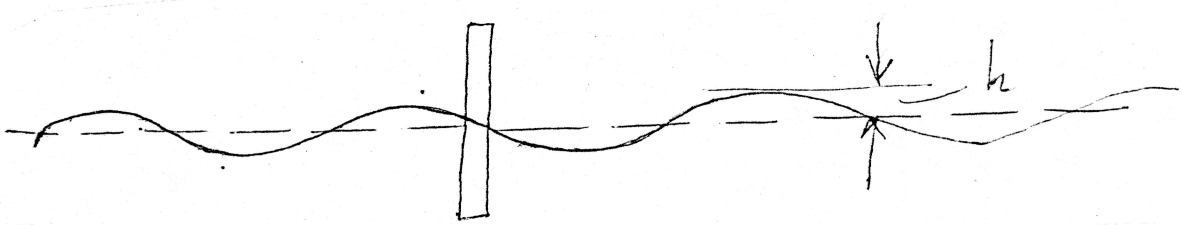
CLASSICAL MECHANICS: Do all four problems.

1. A spaceship is orbiting a planet in a circular orbit. It is given a weak, impulsive radial blow at sometime, as shown in the sketch below.



Find the form of the new orbit, and sketch its shape.

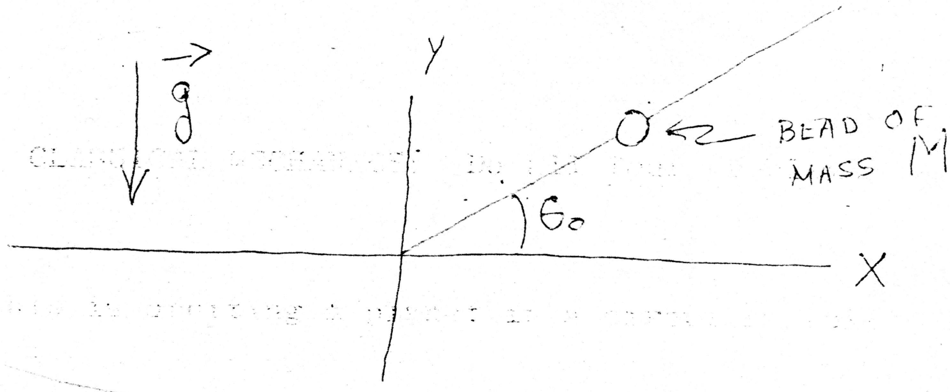
2. A stick of length ℓ floats in water, and gravity acts downward. The density of the water is ρ, and the density of the material of which the stick is made is $\frac{1}{2}\rho$. See sketch.



(a) Calculate the frequency of small oscillations of the stick, if the stick bobs up and down in calm water.

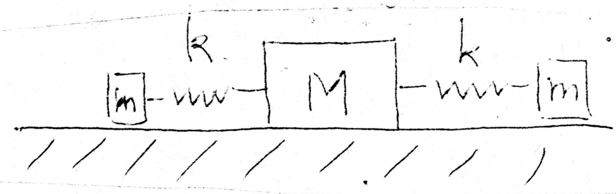
(b) A sinusoidal wave with frequency ω and height h passes by the stick. Calculate the amplitude of the stick as a function of ω, and sketch the result.

3. A bead of mass M slides without friction on a long straight wire inclined at angle θ_o with respect to the x axis. A gravitational field of strength g acts downward, and the wire is rotated about the y direction with angular velocity Ω_o (see sketch on next page).



Solve for the motion of the bead as a function of time, if it is presumed to be located at $x = x_0$ at time zero, with zero velocity along the wire. Describe the subsequent motion of the bead, for various possible values of x_0.

4. Two masses, each of mass m, and a third with mass M are hooked together by identical springs, as shown below. Each spring has force constant k, and the masses sit on a frictionless floor.



(a) Find the normal mode frequencies of the system, and sketch the motion associated with each frequency.

(b) The right hand spring is replaced by one with spring constant $k' > k$. Without performing a calculation, indicate which normal mode frequencies change, and sketch the motion of the masses for each normal mode of the new system. The sketch should emphasize any differences with the sketches in (a), and motivate the sketch with brief comments on the physical reasons why the change occurs as you draw it.

Physics Qualifying Examination UCI May 17, 1976

THERMODYNAMICS
AND
STATISTICAL MECHANICS

Do all 4 problems.

1. The force constant of a delicate spring balance is K $K = 0.5$ N/m ($F = -Kx$). Because of the bombardment by air molecules, an object supported by the spring executes a random vibration.

   a) What is the average energy of this random motion?

   b) What is the rms displacement of the object away from its equilibrium position at room temperature?

   c) The spring balance is used to weigh an object whose mass is approximately 1 gram. What fractional uncertainty in its weight is contributed by the thermal fluctuations?

2. a) Show that for a photon gas, the number of photons depends on the temperature. Derive the form of this equation.

   b) Show that the average energy per photon is proportional to kT, i.e., similar to the classical ideal gas problem. Derive the form of this equation.

3. In a fission reactor operating at 600°K, the free neutron flux near its center is 4×10^{24} neutrons m^{-2} - sec^{-1}.

   a) Find the number of neutrons per cubic meter.

   b) Show that, to a good approximation, one can treat this system of Fermi-Dirac particles with Maxwell-Boltzman statistics.

   c) What is the pressure generated by this neutron gas?

   d) Derive an equation for the Fermi Energy (Chemical Potential) of this gas, and calculate its value.

4. Consider a very long (infinite) hollow pipe of cross sectional area A standing vertically on the surface of the earth. Assume that the gas within the pipe is an ideal monatomic gas of mass m at a constant temperature T.

   a) What is the partition function for this gas?

   b) What is the average energy per molecule?

   c) Does this gas obey the equipartition theorem? Explain your answer.

Physics Qualifying Examination UCI May 17, 1976

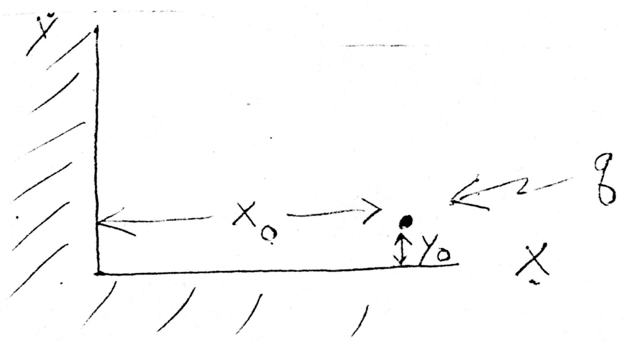
ELECTRICITY AND MAGNETISM

There are three problems. Complete all three.

1. Derive the equation for the resistance/unit length between two cylindrical electrodes of diameter a, separated by the distance ℓ. The electrodes are immersed in a fluid of conductivity σ, and assume $a \ll \ell$.

2. A spherical shell (very thin) has a uniform surface charge density σ spread over its surface. It spins about the z axis with angular velocity Ω_0.
   (a) Find expressions for the electric and magnetic fields everywhere.
   (b) Calculate the Poynting vector, and draw a sketch of of the lines of $\vec{S}$.
   (c) Calculate the total angular momentum density stored in the fields, and the total angular momentum.

3. A charge + q is placed near the corner of two intersecting, grounded and perfectly conducting planes, as shown in the sketch:



Assume that $y_0 \ll x_0$. The charge oscillates in the y direction, with small amplitude $y(t) = \Delta \cos(\omega t)$, where the frequency ω is small, in the sense that $\omega \ll c/x_0$. Find a formula for the Poynting vector far from the corner.

GENERAL PHYSICS

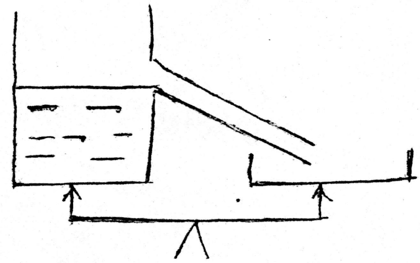
Do 8 out of 10.

1. Give numerical values for
   a) mass of: electron, proton, earth, sun, our galaxy.
   b) size of: classical electron radius, nuclear diameter, first Bohr orbit, distance to sun, distance to nearest star.
   c) time scale for: decay of an excited nucleus, decay of excited atom, half-life of carbon-14, half-life of uranium^{235}, ~~size of earth,~~ age of universe.
   d) energy required to: dissociate a molecule, ionize an atom, disrupt a nucleus, break ~~parton~~ PROTON into its component quarks?

2. Complete the listed nuclear and particle reactions:
   a) $p + e^- \rightarrow \_\_\_\_ + \nu_e$
   b) $\pi^+ \rightarrow \mu^+ + \_\_\_\_$
   c) $\mu^+ \rightarrow \_\_\_\_ + \nu_e + \bar{\nu}_\mu$
   d) $C^{12} + He^4 \rightarrow \_\_\_\_$
   e) $Li^7 + p \rightarrow He^4 + \_\_\_\_$
   f) $C^{13} + d \rightarrow C^{14} + \_\_\_\_$

3. An electron with velocity $\vec{v}$ close to speed of light is injected into a magnetic field of strength $\vec{B}$.
   a) What path will it follow?
   b) What is the emitted radiation called?
   c) In which direction is the radiation emitted?
   d) A fixed observer will see the radiation as a series of pulses. Why?
   e) What is the range of frequencies comprising the radiation?

4. Consider the device shown in the sketch. The tank on the left has a side arm. The water level is at the bottom edge of this arm. Add a bit of ice to the left tank. Is the balance preserved? Explain your answer.



5. The "arrow of time" is said to be related to the existence of irreversible phenomena. Elucidate.

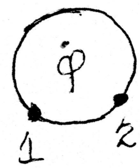
6. a) A stone is thrown from a boat into the swimming pool in which the boat is floating. Does the water level rise, fall, or remain the same? Explain briefly.
b) What happens to the level if a hole is made in the boat and it sinks?

7. A coin with an off center hole is spun. Does the hole migrate to the top, side, or bottom? Explain.

8. What is a boson? A fermion? Give an example of each. Describe the connection with statistics.

9. What is:
a) Olber's paradox? How can it be resolved?
b) The density of a neutron star?
c) Mach's principle?

10. Given a conducting ring threaded by a changing magnetic flux, what is the potential difference between points 1 and 2?



Physics Qualifying Examination UCI May 18, 1976

MATHEMATICAL PHYSICS

1. a) Evaluate the integral $\int_0^\infty \frac{x^2 dx}{x^4+1}$.

b) Evaluate the integral $\int_0^\infty \frac{\ln x}{x^2+b^2} dx$.

(Hint: Make use of the function $\frac{(\ln z)^2}{z^2+b^2}$.)

2. An anharmonic, one-dimensional oscillator is driven by an external cosinusoidal force, so that its equation of motion is

$$m\ddot{x} = -kx - \lambda x^2 + F_o \cos(\omega t)$$

Among the conseqquences of the non-linear term in this equation is that the amplitude x(t) acquires a D.C. component, as well as a component that oscillates at frequency 2ω. Find the expressions for each of these two components, to lowest order in the small quantity λ.

3. A square membrane of side L is confined to a rigid frame, and it vibrates with amplitude $u(\vec{x},t)$ that obeys the equation

$$T\nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} - k \frac{\partial u}{\partial t} = f(t)\delta(x-x_o)\delta(y-y_o) \ .$$

Find a formal expression for the amplitude $u(\vec{x},t)$.

4. A function f(x) is represented in the closed interval [0,1] by the Fourier series

$$f(x) = \sum_{n=-\infty}^{+\infty} f_n \exp[2\pi inx] \ .$$

Under what conditions are the Fourier components of df/dx given by $2\pi in\ f_n$?

PHYSICS QUALIFYING EXAMINATION UCI May 19, 1976

QUANTUM MECHANICS I

Do all the problems.

1. Suppose a parameter α occurs in the Hamiltonian H. It might be e.g. the range of the potential. Show that for a stationary, normalized state ψ that

$$\frac{\partial E}{\partial \alpha} = \frac{\partial}{\partial \alpha} \langle \psi H \psi \rangle = \langle \psi \frac{\partial H}{\partial \alpha} \psi \rangle$$

2. Given the harmonic oscillator Hamiltonian

$$H_o = \frac{p^2}{2m} + \tfrac{1}{2} m \omega^2 x^2$$

The matrix element $x_{n,n+1} = \langle n|x|n+1\rangle = \beta\left(\frac{n+1}{2}\right)^{\frac{1}{2}}$

where $\beta = \left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}$.

a) Calculate $(x^3)_{n,n+1}$ and $(x^3)_{n,n+3}$

b) Given the perturbation $V = \alpha x^3$, calculate the perturbed energy of the oscillator ground state to second order. $\frac{1}{2}\hbar\omega\left(1 - \frac{3}{2\hbar^2}\alpha^2\right)$

3. Calculate a matrix representation of the spin operators $\hat{S}_z$, $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$ for the case S = 2.

4. A positron with velocity v, such that $c \gg v \gg e^2/\hbar$ is incident on a hydrogen atom in the ground state. Calculate the angular scattering cross section for the situation where where the hydrogen atom remains in the ground state. Justify all your approximations.

5. Given that the following squared matrix element

$$\left|\left\langle \alpha, J=\tfrac{3}{2}, J_z=\tfrac{1}{2}\right| Y_{\ell=2}^{m_\ell=0} \left|\beta, J=\tfrac{1}{2}, J_z=\tfrac{1}{2}\right\rangle\right|^2 = 17$$

Calculate

$$\left|\left\langle \alpha, J=\tfrac{3}{2}, J_z=-\tfrac{1}{2}\right| Y_{\ell=2}^{m_\ell=-1} \left|\beta, J=\tfrac{1}{2}, J_z=\tfrac{1}{2}\right\rangle\right|^2$$

and

$$\left|\left\langle \alpha, J=\tfrac{3}{2}, J_z=\tfrac{1}{2}\right| Y_{\ell=2}^{m_\ell=-1} \left|\beta, J=\tfrac{1}{2}, J_z=\tfrac{1}{2}\right\rangle\right|^2$$

6. Prove that trace $\gamma_\mu A^\mu \gamma_\sigma B^\sigma = 4A\cdot B$ where the γ's are the Dirac matrices.

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2	Coefficients		
m_1 m_2			
. .			

1/2 × 1/2

m_1 m_2	J=1, M=+1
+1/2 +1/2	1

m_1 m_2	J=1, M=0	J=0, M=0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	J=1, M=-1
-1/2 -1/2	1

1 × 1/2

m_1 m_2	J=3/2, M=+3/2
+1 +1/2	1

m_1 m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1 -1/2	1/3	2/3
0 +1/2	2/3	-1/3

m_1 m_2	J=3/2, M=-1/2	J=1/2, M=-1/2
0 -1/2	2/3	1/3
-1 +1/2	1/3	-2/3

m_1 m_2	J=3/2, M=-3/2
-1 -1/2	1

2 × 1

m_1 m_2	J=3, M=+3
+2 +1	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+2 0	1/3	2/3
+1 +1	2/3	-1/3

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 -1	1/15	1/3	3/5
+1 0	8/15	1/6	-3/10
0 +1	6/15	-1/2	1/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1 -1	1/5	1/2	3/10
0 0	3/5	0	-2/5
-1 +1	1/5	-1/2	3/10

m_1 m_2	J=3, M=-1	J=2, M=-1	J=1, M=-1
0 -1	6/15	1/2	1/10
-1 0	8/15	-1/6	-3/10
-2 +1	1/15	-1/3	3/5

m_1 m_2	J=3, M=-2	J=2, M=-2
-1 -1	2/3	1/3
-2 0	1/3	-2/3

m_1 m_2	J=3, M=-3
-2 -1	1

1 × 1

m_1 m_2	J=2, M=+2
+1 +1	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+1 0	1/2	1/2
0 +1	1/2	-1/2

m_1 m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1 -1	1/6	1/2	1/3
0 0	2/3	0	-1/3
-1 +1	1/6	-1/2	1/3

m_1 m_2	J=2, M=-1	J=1, M=-1
0 -1	1/2	1/2
-1 0	1/2	-1/2

m_1 m_2	J=2, M=-2
-1 -1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta \, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}} \, Y_\ell^m e^{-im\phi}$$

3/2 × 1

m_1 m_2	J=5/2, M=+5/2
+3/2 +1	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	-2/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2 -1	1/10	2/5	1/2
+1/2 0	3/5	1/15	-1/3
-1/2 +1	3/10	-8/15	1/6

m_1 m_2	J=5/2, M=-1/2	J=3/2, M=-1/2	J=1/2, M=-1/2
+1/2 -1	3/10	8/15	1/6
-1/2 0	3/5	-1/15	-1/3
-3/2 +1	1/10	-2/5	1/2

m_1 m_2	J=5/2, M=-3/2	J=3/2, M=-3/2
-1/2 -1	3/5	2/5
-3/2 0	2/5	-3/5

m_1 m_2	J=5/2, M=-5/2
-3/2 -1	1

2 × 1/2

m_1 m_2	J=5/2, M=+5/2
+2 1/2	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 -1/2	1/5	4/5
+1 +1/2	4/5	-1/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1 -1/2	2/5	3/5
0 +1/2	3/5	-2/5

m_1 m_2	J=5/2, M=-1/2	J=3/2, M=-1/2
0 -1/2	3/5	2/5
-1 +1/2	2/5	-3/5

m_1 m_2	J=5/2, M=-3/2	J=3/2, M=-3/2
-1 -1/2	4/5	1/5
-2 +1/2	1/5	-4/5

m_1 m_2	J=5/2, M=-5/2
-2 -1/2	1

3/2 × 1/2

m_1 m_2	J=2, M=+2
+3/2 +1/2	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+3/2 -1/2	1/4	3/4
+1/2 +1/2	3/4	-1/4

m_1 m_2	J=2, M=0	J=1, M=0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	J=2, M=-1	J=1, M=-1
-1/2 -1/2	3/4	1/4
-3/2 +1/2	1/4	-3/4

m_1 m_2	J=2, M=-2
-3/2 -1/2	1

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J - j_1 - j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2 × 3/2

m_1 m_2	J=3, M=+3
+3/2 +3/2	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	-1/2

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2 -1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	-2/5
-1/2 +3/2	1/5	-1/2	3/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+3/2 -3/2	1/20	1/4	9/20	1/4
+1/2 -1/2	9/20	1/4	-1/20	-1/4
-1/2 +1/2	9/20	-1/4	-1/20	1/4
-3/2 +3/2	1/20	-1/4	9/20	-1/4

m_1 m_2	J=3, M=-1	J=2, M=-1	J=1, M=-1
+1/2 -3/2	1/5	1/2	3/10
-1/2 -1/2	3/5	0	-2/5
-3/2 +1/2	1/5	-1/2	3/10

m_1 m_2	J=3, M=-2	J=2, M=-2
-1/2 -3/2	1/2	1/2
-3/2 -1/2	1/2	-1/2

m_1 m_2	J=3, M=-3
-3/2 -3/2	1

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2} \qquad d^1_{0,0} = \cos\theta$$

2 × 3/2

m_1 m_2	J=7/2, M=+7/2
+2 +3/2	1

m_1 m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	-3/7

m_1 m_2	J=7/2, M=+3/2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 -1/2	1/7	16/35	2/5
+1 1/2	4/7	1/35	-2/5
0 3/2	2/7	-18/35	1/5

m_1 m_2	J=7/2, M=+1/2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+2 -3/2	1/35	6/35	2/5	2/5
+1 -1/2	12/35	5/14	0	-3/10
0 1/2	18/35	-3/35	-1/5	1/5
-1 3/2	4/35	-27/70	2/5	-1/10

m_1 m_2	J=7/2, M=-1/2	J=5/2, M=-1/2	J=3/2, M=-1/2	J=1/2, M=-1/2
+1 -3/2	4/35	27/70	2/5	1/10
0 -1/2	18/35	3/35	-1/5	-1/5
-1 1/2	12/35	-5/14	0	3/10
-2 3/2	1/35	-6/35	2/5	-2/5

m_1 m_2	J=7/2, M=-3/2	J=5/2, M=-3/2	J=3/2, M=-3/2
0 -3/2	2/7	18/35	1/5
-1 -1/2	4/7	-1/35	-2/5
-2 1/2	1/7	-16/35	2/5

m_1 m_2	J=7/2, M=-5/2	J=5/2, M=-5/2
-1 -3/2	4/7	3/7
-2 -1/2	3/7	-4/7

m_1 m_2	J=7/2, M=-7/2
-2 -3/2	1

2 × 2

m_1 m_2	J=4, M=+4
+2 +2	1

m_1 m_2	J=4, M=+3	J=3, M=+3
+2 +1	1/2	1/2
+1 +2	1/2	-1/2

m_1 m_2	J=4, M=+2	J=3, M=+2	J=2, M=+2
+2 0	3/14	1/2	2/7
+1 1	4/7	0	-3/7
0 2	3/14	-1/2	2/7

m_1 m_2	J=4, M=+1	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 -1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	-1/14	-3/10
0 1	3/7	-1/5	-1/14	3/10
-1 2	1/14	-3/10	3/7	-1/5

m_1 m_2	J=4, M=0	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+2 -2	1/70	1/10	2/7	2/5	1/5
+1 -1	8/35	2/5	1/14	-1/10	-1/5
0 0	18/35	0	-2/7	0	1/5
-1 1	8/35	-2/5	1/14	1/10	-1/5
-2 2	1/70	-1/10	2/7	-2/5	1/5

m_1 m_2	J=4, M=-1	J=3, M=-1	J=2, M=-1	J=1, M=-1
+1 -2	1/14	3/10	3/7	1/5
0 -1	3/7	1/5	-1/14	-3/10
-1 0	3/7	-1/5	-1/14	3/10
-2 1	1/14	-3/10	3/7	-1/5

m_1 m_2	J=4, M=-2	J=3, M=-2	J=2, M=-2
0 -2	3/14	1/2	2/7
-1 -1	4/7	0	-3/7
-2 0	3/14	-1/2	2/7

m_1 m_2	J=4, M=-3	J=3, M=-3
-1 -2	1/2	1/2
-2 -1	1/2	-1/2

m_1 m_2	J=4, M=-4
-2 -2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. (Table extended April 1974.)

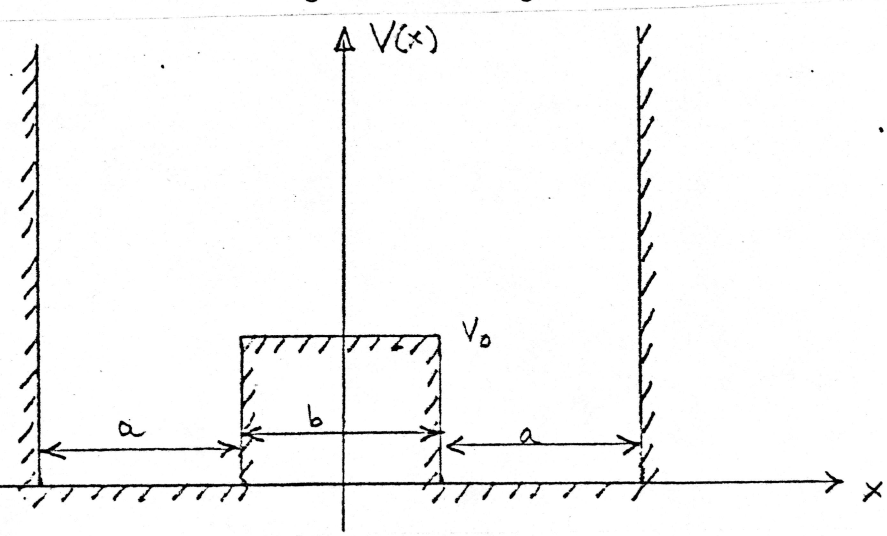
QUANTUM MECHANICS II

Do all the problems.

1. An electron is in the ground state of tritium, ^3H. A beta decay reaction, $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$ occurs, changing the ^3H nucleus to ^3He. Calculate the probability that the atomic electron will be in the ground state of $^3He^+$. (Hint: as an approximation, ignore the interaction between the beta decay electron and the atomic electron.)

2. Calculate the Zeeman structure and sketch the energy levels of an atom in the $D_{5/2}$ state which is immersed in a weak magnetic field.

3. Given the potential energy shown, sketch the wave functions and energy levels for $E < V_o$ and $E > V_o$.



Discuss the effect on adjacent energy levels as either V_o or b becomes very large.

4. The eigenfunctions of spin $\frac{1}{2}$ are α and β which are the usual up and down eigenfunctions of the z component of the spin operator, i.e., $\hat{S}_z$. Given a system of 3 spin $\frac{1}{2}$ particles, what are the eigenfunctions of spin 3/2? Give your answer in terms of the α's and β's mentioned above.

Physics Qualifying Examination UCI September 20, 1976

QUANTUM MECHANICS I

(Open Book)

Do all problems

Problem 1:

Consider a system with angular momentum $\hat{J}$ and gyromagnetic ratio g_J which is at rest in a uniform magnetic field $\vec{B}$.

(a) What is the interaction Hamiltonian?

(b) Using the quantum mechanical equation for the time variation of the operator $\hat{J}$, find the equation of motion for $\hat{J}$.

(c) How does the result in (b) differ from the corresponding classical equation of motion?

Problem 2:

Calculate a matrix representation of the angular momentum operators $\hat{J}_z$, $\hat{J}_\pm \equiv \hat{J}_x \pm i\hat{J}_y$ for the case $j = 3/2$.

Problem 3:

A particle of mass m is in the ground state of a one dimensional potential, $V(x) = \frac{1}{2} k x^2$. The potential disappears instantaneously so that the particle is free.

(a) What is the probability that the particle has momentum in the range $(p, p + dp)$?

(b) After the removal of the potential, the energy of the particle is $p^2/2m$, which need not be equal to the ground state energy. What is the explanation for the apparent lack of energy conservation?

Problem 4:

Two different particles of mass m interact by means of an attractive potential which takes on the value $-V_o$ for interparticle separation less than a, and vanishes for separation greater than a.

(a) Calculate, in first Born approximation, the scattering amplitude for non-relativistic scattering.

Problem 4 (Cont'd.)

(b) Find the dependence of the scattering amplitude on the depth and volume of the potential well in the limit of vanishing kinetic energy.

(c) State the optical theorem for the total cross section and apply it to the scattering amplitude calculated in part (a). What is wrong and why?

Problem 5:

Consider the hydrogen atom, and assume that the proton, instead of being a point source of the coulomb field, is a uniformly charged sphere of radius R, where $R \ll a_o$.

(a) What is the resulting coulomb potential?

(b) Calculate the energy shift for the ground state.

Problem 6:

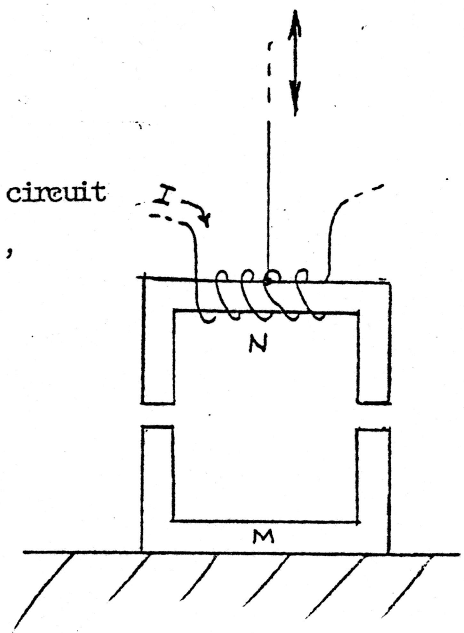
Show that when the electromagnetic field is non-zero, the solutions of the Dirac equation satisfy a second order differential equation which differs from the Klein-Gadon equation.

Also show that the Dirac theory correctly predicts the magnitude of the intrinsic magnetic moment of the electron (i.e. $g_s = 2$).

ELECTRICITY AND MAGNETISM

DO ALL PROBLEMS

Problem 1:

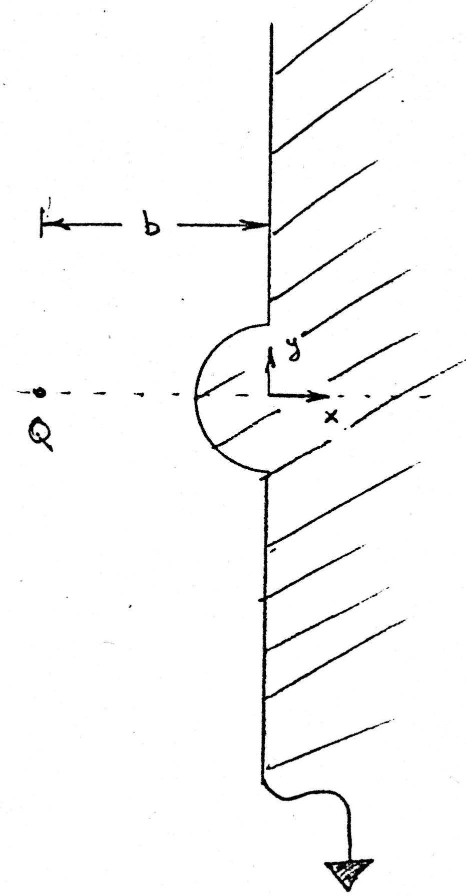


You are given the electro-magnetic "horseshoe" circuit shown, in which each piece has permeability $\mu \gg \mu_0$, cross-sectional area A, and length $\ell \gg A^{\frac{1}{2}}$. The winding has N turns.

The upper yoke is suspended from a hoist and can be positioned over the lower yoke which is sitting on the ground.

If the mass of the lower piece is M, what minimum current I_m will allow it to be raised by the hoist?

Problem 2:



Consider the grounded conductor shown, which has the form of a plane with a hemispherical bump of radius a.

(a) If a charge Q is placed a distance b to the left of the plane, on the axis of symmetry, as shown, find the potential everywhere.

(b) Indicate, giving general but specific formulae, how you would find the force on Q.

Problem 3:

(a) Take a round wire of radius a and frequency independent conductivity σ. What is the ratio of the resistance at (very) high frequencies to that at D.C.?

(b) Consider a plane slab of dielectric with constant ϵ and thickness d in vacuum, and normally propagating plane waves of frequency ω. What is the effective wave impedance [$\hat{E}/\hat{H}$, where the hats denote amplitudes] looking into the front surface of the slab?

Problem 4:

Consider a cylindrical waveguide of inner radius a with an applied axial magnetic field B_o, and filled with a uniform, cold, stationary, electron plasma ($n_e = n_o$, $T_e \approx 0$, $m_i \approx \infty$). Derive and plot the dispersion relation for the simplest, cylindrically symmetric, slow ($v_p \ll c$), small-amplitude mode which propagates in this guide. Define all of the terms used.

NOTE: Treat the plasma as a dielectric and ignore the wave magnetic field term in Faraday's law.

Physics Qualifying Examination UCI September 21, 1976

CLASSICAL MECHANICS

Do all Problems

Problem 1:

A latter rests against a smooth, vertical wall and slides without friction on wall and floor. What is the equation of motion assuming that the ladder maintains contact with the wall? If initially the ladder is at rest at an angle α with the floor, at what angle, if any, will it leave the wall?

Problem 2:

A rocket moves with initial velocity v_o toward the moon of mass M, radius r_o. Find the cross section σ for striking the moon. Take the moon to be at rest, and ignore all other bodies.

Discuss briefly the limiting cases (1) where the attraction goes to zero and (2) as v_o becomes very large.

Problem 3:

A mass m is hung from a fixed support by a spring of constant k whose relaxed length is $\ell = 2\,mg/k$. A second equal mass is hung from the first mass by an identical spring. Each spring exerts a force only along the line joining its two ends, but <u>may pivot freely in any direction</u> at its ends. Find the normal coordinates, and the corresponding frequencies, for small vibrations of this system from its equilibrium position. Neglect the mass of the springs.

Give a brief qualitative description of the motion.

Problem 4:

Show that the axis of rotation of a freely rotating symmetrical rigid body precesses in space with an angular velocity

$$\Omega = (2\beta + \beta^2 + \sec^2 \alpha_b)\,\omega_3$$

where the axes are numbered 1, 2, 3. The 3-axis is the axis of symmetry. The moments of inertia about the axes are I_1, I_2, I_3, and

$$\beta = \frac{I_3 - I_1}{I_1}$$

The angle α_b is the angle between the axis of symmetry and the vector angular velocity, $\vec{\omega}$, of the rotating body.

Give a brief qualitative description of the motion. A simple sketch may be useful.

Physics Qualifying Examination UCI, September 21, 1976

QUANTUM MECHANICS II

(Closed Book)

Do all problems

Problem 1:

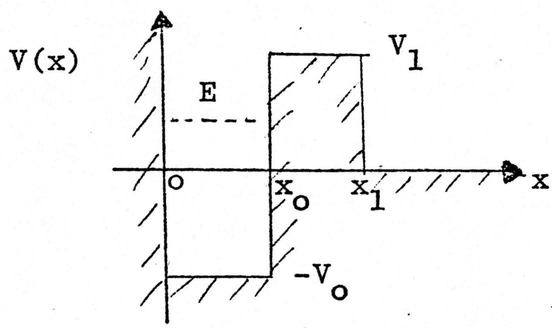
The radius of a Si nucleus ($Z = 14$, $N = 14$) is about 3.6×10^{-13} cm. What is the probability of finding a μ^- inside the nucleus if the μ^- is in the ground state?

Problem 2:

Show how the invariance of the Hamiltonian under some kind of transformation is in general connected to a conservation law.

Problem 3:

Consider the one-dimensional potential shown below



Estimate the lifetime of a particle of mass m and energy E ($E < V_1$) in this potential.

Problem 4:

Two of the low lying excited states of sodium ($Z = 11$) are $^2P_{1/2}$ and $^2P_{3/2}$. The difference in energy of these states is ΔW where the $^2P_{1/2}$ level is higher than the $^2P_{3/2}$ level. (Ground state configuration of the Na electrons is $1s^2$, $2s^2$, $2p^6$, $3s^1$).

(a) Sketch and identify the quantum numbers of the resulting levels as a function of an external magnetic field, $\vec{B}$, from the low field limit through to the high field limit.

(b) What value of the magnetic field separates the high and low field limits?

THERMODYNAMICS
AND STATISTICAL MECHANICS

Answer 4 of the following 5 questions. If you attempt all 5, indicate clearly which one you do not want graded.

Problem 1:

A wall galvanometer is disconnected from any electrical circuits and allowed to swing freely. It is found to have a period of 30 seconds and upon further observation it is found that it never comes to rest. If the moment of inertia is 10 g-cm, what is the mean amplitude of the eventual motion? What is the cause of this motion?

Problem 2:

The energy of a particular system of N identical particles is given by

$$E = \frac{1}{2m} \sum_{i=1}^{N} P_i^2$$

where m is the particle mass and P_i is the momentum of the i^{th} particle. The particles are confined to a volume V.

(a) Using classical methods, calculate the entropy of this system.

(b) Show that your result in (a) is not a satisfactory expression for entropy on very general grounds and make a correction which resolves the problem.

(Note: The difficulty referred to in this problem is known as the Gibbs Paradox.)

Problem 3:

Consider a substance which has the following two properties:

i) At a specified constant temperature, T_o, the work done by it on expansion from volume V_o to volume V is

$$W = R T_o \ln(V/V_o)$$

ii) The entropy is given by

$$S = R\frac{V_o}{V}\left(\frac{T}{T_o}\right)^{\alpha}$$

where V_o , T_o , and α are fixed constants.

Problem 3 (Cont'd.)

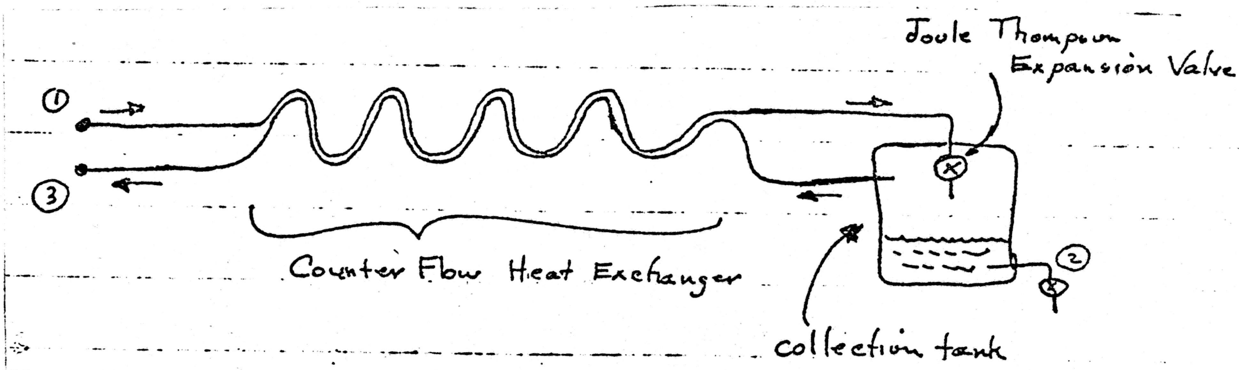
(a) Calculate the Helmholtz free energy.

(b) Find the equation of state.

(c) Find the work done during isothermal expansion at temperature T (T not equal to T_o).

Problem 4:

Air can be liquified by passing it through the apparatus which is sketched below:



Air is compressed to a pressure P_1, cooled to room temperature, T_1, and introduced into the apparatus at position marked (1). It is cooled by the returning air in the heat exchanger and then expands irreversibly through the Joule-Thompson valve. The vapor which does not condense passes into the return line of the heat exchanger and out of the system at position 3 where it has pressure P_3 and temperature T_3. For an ideal heat exchanger $T_3 = T_1$,

(a) What thermodynamic function is conserved during the expansion process? Explain why.

(b) During steady state operation a constant fraction, ε, of the vapor passing into the system condenses and is drawn off as liquid at position 2. Set up a balance equation for the conserved quantity and solve for ε in terms of the values of the thermodynamic function at positions 1, 2, and 3.

(c) Discuss why the gas is cooled to room temperature after compression and before introduction into this apparatus.

Problem 5:

A simple model of a rubber-band is a chain of N segments ($N >> 1$) joined end to end. Each segment has a fixed length ℓ and can be either parallel or antiparallel to the axis of the rubber-band. Stretching the rubber-band tends to pull the links into parallelism. Thermal agitation randomizes their directions. Use this model to find an expression for the length, L, of the rubber-band as a function of temperature, T, and applied force, F. Discuss the relation of this model to that of a paramagnetic salt in a magnetic field.

Physics Qualifying Examination UCI September 22, 1976

MATHEMATICAL PHYSICS

Problem 1:

Evaluate the integral

$$\int_0^\infty \frac{x \sin x}{x^2 + a^2}\, dx \quad , \quad a > 0$$

Problem 2:

Find a function w(z) which realizes a conformal mapping of the half-disc $|z| < 1$, $\mathrm{Im}\, z > 0$ onto the upper half-plane $\mathrm{Im}\, w > 0$, and such that: $w(0) = -1$, $w(-1) = 0$, $w(+1) = \infty$.

Problem 3: (b) and (c) may be solved independently of (a).

(a) Show that the equation describing the small oscillations (in a vertical plane which rotates together with the string) of a homogeneous perfectly flexible string of length ℓ rotating around a vertical axis through one of its ends with angular velocity ω, is:

$$\frac{\partial}{\partial x}\left[(\ell^2 - x^2)\frac{\partial u}{\partial x}\right] - \frac{2}{\omega^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

with boundary conditions:

$$u(0,t) = 0 \quad u(\ell,t) = \text{finite} \qquad (2)$$

(b) Find the normal modes of Eq. (1) with boundary conditions (2).

(c) Determine the solution corresponding to initial conditions $u(x,0) = f(x)$, $u_t(x,0) = g(x)$.

Problem 4:

Show that the harmonic oscillator wave functions (Weber-Hermite functions)

$$\Psi_n(x) = (-1)^n (2^n n! \sqrt{\pi})^{-\frac{1}{2}} e^{\frac{x^2}{2}} \frac{d^n}{dx^n} e^{-x^2}$$

(Continued)

Problem 4 (Cont'd.)

(solutions of the equation

$$\frac{d^2\Psi}{dx^2} - x^2\,\Psi + (2n+1)\,\Psi = 0)$$

are, up to a constant $(-i)^n$, their own Fourier transforms:

$$(2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \Psi_n(x)\, e^{-ixu}\, dx = (-i)^n\, \Psi_n(u) \quad .$$

Physics Qualifying Examination UCI September 22, 1976

GENERAL PHYSICS

Do all Problems

Problem 1:

Discuss in two or three sentences the present evidence for two of the following:

(a) The existence of a new internal quantum number ("charm") associated with elementary particles.

(b) The existence of astronomical "black holes".

(c) The superfluid behavior of He^3.

Problem 2:

Give approximate numerical values for the following:

The radius of:
- a proton
- a hydrogen atom
- the moon
- the earth's orbit
- our galaxy
- the "visible" universe

The lifetimes of:
- a ρ meson
- a π^+ meson
- a typical atomic excited state
- a neutrino

The age of:
- the earth
- the universe(?)

Problem 3:

Use dimensional analysis to find approximate expressions for the following quantities. You must decide what parameters may be needed in each expression. It is sufficient to guess an expression and demonstrate that it is dimensionally correct. Indicate in which cases (if any) the expression you find is not unique.

(a) The force exerted by a wind on a wall perpendicular to its motion.

(b) The space charge limited current in a vacuum tube diode.

(c) The time it would take the universe to collapse, if it were initially neither expanding nor contracting.

(d) The "radius" of a black hole.

Problem 4:

List the four fundamental forces observed in nature, and indicate:

(a) their relative strengths and range,

(b) the internal and/or space time symmetries (if any) violated by each, and

(c) examples of natural phenomena governed by each.

Problem 5:

Estimate the number of driver's license road test examiners employed in the State of California, outlining your reasoning.

Problem 6:

A metallic cathode illuminated by light of frequency ν will emit electrons (photoelectric effect) which may be collected by a metal plate so long as the potential externally applied between the cathode and plate does not exceed a "stopping potential" given by

$$V = h\nu - \varphi_p$$

where φ_p is the work function for the collecting plate (not, as often stated in textbooks, for the emitting cathode).

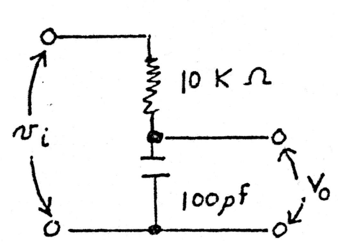
Show that the above expression is correct. (Hint: consider contact potentials.)

Problem 7:

Water is poured into a barrel at the rate of 120 lb/minute from a height of 16 feet. The barrel weighs 25 lb and rests on a scale. Find the scale reading after the water has been pouring into the barrel for one minute.

Problem 8:

Consider the RC circuit shown, with an input sine wave of amplitude v_i and output of amplitude v_o.



(a) For all frequencies, find the ratio v_o/v_i.

(b) Qualitatively plot the phase angle versus frequency.

(c) For what frequency range does the circuit act as a filter? Explain.

(d) For what frequency range does integration occur? Explain.

Problem 9:

Given a sound source with velocity V_s and a listener moving with velocity V_o colinearly with the source, calculate the frequency heard by the listener.

Problem 10:

Calculate the minimum kinetic energy a proton must have in order to produce an antiproton by the reaction $pp \rightarrow ppp\bar{p}$ (target proton at rest).

Physics Qualifying Examination September 26, 1977

MATHEMATICAL PHYSICS

(3 hours)

Closed Book.
Do all problems.

1\. a. Define and give an example of the following: entire function, branch point, non-isolated essential singu-
(15) larity, saddle point.

b. Evaluate the integral

(15)
$$\int_0^\infty \frac{\log (a^2 + x^2)}{a^2 + x^2}\, dx$$

2\. Polynomials of a certain kind satisfy the differential equation

$$xL_n''(x) + (1 - x)L_n'(x) + nL_n(x) = 0$$

They have the following contour integral representation:

$$L_n(x) = \frac{1}{2\pi i} \oint \frac{e^{-\frac{xz}{1-z}}}{(1-z)z^{n+1}}\, dz$$

where the contour of integration is a closed curve that encloses the origin and does not enclose $z = 1$.

(10) a. Find the generating function $\sum_{n=0}^{\infty} L_n(x)z^n$

b. Derive the recurrence relations

(10)
$$(n+1)L_{n+1}(x) = (2n+1 - x)L_n(x) - nL_{n-1}(x)$$

(10)
$$x\, L_n'(x) = nL_n(x) - nL_{n-1}(x)$$

3. The Green's function $G(\underset{\sim}{x})$ satisfies

$$(\nabla^2 - k^2)G(\underset{\sim}{x}) = -\delta(\underset{\sim}{x})$$

and the boundary condition $\lim_{|\underset{\sim}{x}| \to \infty} G(\underset{\sim}{x}) = 0$

a. Find solutions in one, two, and three dimensions which correspond to a plane source, line source and point
(24) source respectively.

b. By considering the relation between a point source and
(8) a line source, show that

$$\int_{r=\rho}^{\infty} \frac{e^{-kr}}{\sqrt{r^2 - \rho^2}}\, dr = K_o(k\rho)$$

c. By considering the relation between a line source and
(8) a plane source show that

$$\int_{\rho=x}^{\infty} \frac{K_o(k\rho)\,\rho d\rho}{\sqrt{\rho^2 - x^2}} = \frac{\pi}{2}\,\frac{e^{-k|x|}}{k}$$

Useful Relations:

$$J_o(x) = \frac{1}{2\pi}\int_o^{2\pi} d\varphi\, e^{ix\cos\varphi}$$

$$K_o(x) = \int_o^{\infty} y\, dy\, \frac{J_o(y)}{x^2 + y^2}$$

QUANTUM MECHANICS I (Quantitative)

($4\frac{1}{2}$ hours)

Open Book:
One standard reference text allowed.

1. A particle is known to be localized in the left half of a box (infinite square well) with sides at $x = \pm a$. If all values of x in the left half side are equally probable, a) what wave function describes the particle at t=o? b) Will the particle remain localized at later times? c) Calculate the probability that an energy measurement yields the ground state energy; d) the energy of the first excited state.

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2. Two particles of mass m are placed in an infinite square well potential of side a in the lowest energy state of the system compatible with the conditions below. Assuming that the particles interact with each other according to the potential $V=V_o a\delta(x_1-x_2)$, use first order perturbation theory to calculate the energy of the system under the following conditions:

   a) particles not identical
   b) identical particles of spin 0
   c) identical particles of spin $\frac{1}{2}$ with spins anti-parallel
   d) identical particles of spin 1/2 with spins parallel.

   Also, give the unperturbed wave function for the system in each case.

3. The weakness of gravitational compared to electrostatic interaction is dramatically illustrated by considering a system of two neutrons under the sole influence of their mutual gravitational attraction. The gravitational potential is

$$V_G(r) = -\frac{Gm_n^2}{r} \qquad V_{E\&M}(r) = -\frac{e'^2}{r}$$

where G(=6.67 x 10^{-11}N-m^2-kg^{-2}) is the gravitational constant, and m_n(=1.6748 x 10^{-27}kg) is the neutron mass.

a) Give expressions for the bound state energies and for the "Bohr radius" for such a system.

b) Estimate, to the nearest power of ten (order of magnitude) the numerical value of the ground state energy (in electron volts), and of the Bohr radius (in cm).

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4. Consider two relativistic particles of equal mass μ, moving along the z-direction, and interacting via a one dimensional Coulomb potential. (The "hydrogen" atom in one dimension.). The Hamiltonian of the system is given as (in units of C=1):

$$H = \sqrt{P_1^2 + \mu^2} + \sqrt{P_2^2 + \mu^2} + \gamma|x_1 - x_2| \ ; \ \gamma > 0$$

a) Make the transformation to the center of mass and relative coordinates (R,P) and (x,k). Is P a constant? Why?

b) Plot the classical closed orbits in phase space (x,k space) for a given energy E and center of mass momentum P=0. Discuss the motion as given in your diagram.

c) Apply the WKB method of quantization to find the approximate quantized energy levels for fixed P=0. What is the effective mass (M) of the total system ($M^2 = E^2 - P^2$)? Give your answers in the form of an integral. Evaluate the integral for μ = 0.

5. The new ψ-particle spectroscopy can be described by a "charmonium" model. The assumptions of this model are that spin $\frac{1}{2}$ quarks c (charge 2/3) and $\bar{c}$ (charge -2/3) are bound together strongly and:

   a) The $c\bar{c}$ levels can be obtained from the non-relativistic solutions to the Schrödinger equation for an e'^2/r type potential ($e' \neq e$).

   b) The p-levels have "fine" structure due to a spin-orbit term given by $(\vec{L} \cdot \vec{S})$ in the potential.

   c) The spin-spin forces are small.

   d) In the absence of the spin-orbit term, there is a small perturbation term that would put the 2p levels between the 1s and 2s levels.

   e) The ψ (3100 MeV) is to be identified with the 1^3S_1 level, the ψ' (3700 MeV) is to be identified with the 2^3S_1 level.

   Evaluate from this model:

   1) The mass of the charmed quark.

   2) The coupling strength as an "effective fine structure constant" $\alpha' = \frac{e'^2}{\hbar c}$.

   3) The effective radius of the ψ-meson in cm.

   4) The relative splittings of the P levels due to the $(\vec{L} \cdot \vec{S})$ coupling.

   5) The 2^3S_1 (ψ') shows strong radiative transitions to levels at 3552, 3508, 3415 MeV. Evidence suggests that $\chi(3552)$ has $J \geq 2$ and $\chi(3415)$ has J=0. Is this consistent with 4)?

6. A scattering experiment is carried out where positrons are scattered through the Coulomb interaction from neutral atoms of the element with Z protons and Z electrons with a point nucleus:

  i) What is the Hamiltonian for this problem?

  ii) Discuss briefly the Hartree-Fock approximation for the treatment of the atomic Hamiltonian.

  iii) Show that the scattering cross section is given in first Born approximation by

$$\frac{d\sigma_{fi}}{d\Omega} = \frac{4Z^2}{\left(\underset{\sim}{q}^2\right)^2} \left(\frac{e^2}{\hbar c}\right)^2 \left(\frac{m_e c}{\hbar}\right)^2 \left(\frac{k_f}{k_i}\right) \left| F_{fi}(\underset{\sim}{q}^2) - \delta_{fi} \right|^2 .$$

Here $\underset{\sim}{k}_i, \underset{\sim}{k}_f$ are the initial and final positron wave number; $\underset{\sim}{q} \equiv \underset{\sim}{k}_f - \underset{\sim}{k}_i$ is the three-momentum transfer; the subscripts i, f refer to the initial and final atomic states (with angular momentum quantized along the $\underset{\sim}{q}$ axis); and F_{fi} is the atomic form factor

$$F_{fi}(\underset{\sim}{q}^2) = \frac{1}{Z} \int e^{-i\underset{\sim}{q}\cdot\underset{\sim}{x}} \left\langle f \,\middle|\, \hat{\rho}(\underset{\sim}{x}) \,\middle|\, i \right\rangle d\underset{\sim}{x} .$$

where $\hat{\rho}(\underset{\sim}{x})$ is the atomic charge density operator.

Physics Qualifying Examination UCI September 27, 1977

CLASSICAL MECHANICS

(3 hours)

Open Book: One standard reference text allowed.
Answer any 5 problems. All problems have equal weight.

1. A particle under the influence of a central force moves along a spiral trajectory described in polar coordinates by $r = e^{a\theta}$, with the origin at the center of force. Derive the form of the force law.

2. A beam of particles is scattered by a central potential $V(r) = A/r^n$, $(n > 0)$. Consider the limiting situation in which the deflection is small, and derive an expression for the form of the cross-section for small-angle scattering. In particular obtain the form of the dependence of $\frac{d\sigma}{d\theta}$ on momentum and angle θ as $\theta \to 0$.

3. A mass, m, under the influence of gravity, is hung from a point on the edge of a horizontal circular platform of radius a, by means of a massless rigid rod of length ℓ. The rod is constrained so that it can swing only in a radial direction with respect to the platform. The platform is rotated with a small, constant angular velocity ω, about a vertical axis through its center.

   a. Write the Lagrangian for the system in appropriate coordinates;
   b. Find the equation of motion;
   c. Obtain the solution for the motion of the mass, m, assuming only small displacements from equilibrium.

4. A particle of mass m undergoes orbital motion in a circle of radius a under the influence of a central potential of form $V(r) = Ar^n$. Suppose the particle is slightly perturbed and executes small radial oscillations about the circular orbit.

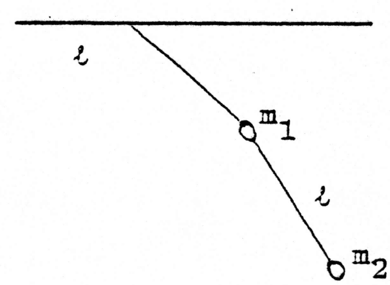
   a. Find the frequency of the radial oscillations and compare it to the orbital frequency.

   b. What conditions must A and n satisfy for such motion to be possible?

5. Two particles of masses m_1 and m_2 are subject to non-uniform potentials V_1 and V_2, respectively, which differ by a constant factor: $V_2/V_1 = \beta$. Find a relation between the times t_1 and t_2 which the two particles take to traverse the same path. State your reasoning clearly.

6. Two particles of masses m_1 , m_2 are suspended from massless rods of equal length ℓ, to form a double pendulum (see Figure). Assume motion is confined to a plane. Find the frequencies and normal modes of the system undergoing small oscillations.

<u>Figure</u>:



Physics Qualifying Examination UCI September 27, 1977

ELECTRICITY AND MAGNETISM

(3 hours)

Open Book: One standard reference text allowed.

Do 4 of the 6 problems.

1. Consider an ionization chamber bounded on opposite sides by two parallel conducting plates of area A and separation d, differing in potential by V_o. Ionization produces electron-ion pairs uniformly through the volume at a rate R m^{-3} sec^{-1}. Now suppose that the electrons are collected in a negligible time by the positive plate, but that the ions drift slowly with velocity $\vec{V} = \mu\vec{E}$, where μ is their mobility. Determine (neglecting edge effects and ion recombination) steady state expressions for the following quantities as a function of position in the chamber.
   a) $V(x)$
   b) $E(x)$
   c) $\rho_+(x)$ (charge density of positive ions)
   d) $J_+(x)$ (current density of positive ions)

2. A thin conducting disk of thickness h, diameter D, and conductivity σ is placed in a uniform alternating magnetic field $B = B_o \sin \omega t$ parallel to the axis of the disk. Neglecting radiation and the self-interaction of currents in the disk, find expressions for:
   a) The average power dissipated in the disk.
   b) The total magnetic field, at distances r >> D away from the disk.
   c) The conditions that h, D, σ, B_o and ω must satisfy in order to justify the neglect of current self interaction.

3. Fact: The equipotential surfaces associated with two parallel infinite line charges having opposite charge densities $+\lambda$ and $-\lambda$ coulombs/meter and separated by a distance $2a$, are circular cylinders of radius R with axes at a distance x from the midpoint between line charges:

$$R = a\ \mathrm{csch}(2\pi\epsilon_o V/\lambda)$$

$$x = a\ \coth(2\pi\epsilon_o V/\lambda)$$

   a) Determine the capacitance per unit length of a system consisting of an infinite conducting cylinder of radius R, with axis a distance d away from a parallel infinite conducting plane.

   b) Determine the charge density distribution on the plane in terms of the linear charge density on the cylinder.

4. Consider the following solution to the energy crisis: A charge Q is uniformly distributed over the surface of a spherical shell of radius R. An electric dipole $\vec{P}$ is made to approach the sphere from a large distance, oriented radially so that it is drawn by the field gradient, doing extractable work. When it reaches the surface of the sphere, it passes through a small hole into the interior field-free region, where its direction is reversed at no cost in energy. It is then removed to a large distance, now oriented so that again the field gradient is doing usable work on the dipole. At a remote point its direction is again reversed, and the procedure cycled repeatedly.

   a) Determine the work done on the dipole in approaching the sphere from infinity.

   b) Find the flaw in the scheme, and quantitatively account for the missing energy.

5. A charge of mass m and charge q is subject to a Newtonian friction. Frictional force

$$= - \gamma v \ ,$$

where γ = constant, v = instantaneous velocity. Upon it is incident a circularly polarized electromagnetic plane wave of intensity I and angular frequency ω in the z direction.

a) Calculate (in the steady state) the rate at which energy is absorbed from the wave by the particle.

b) Calculate directly (i.e., by use of the Lorentz force equation) the rate at which angular momentum is absorbed.

c) How much angular momentum is absorbed per absorbed photon?

6. A radiating quadrupole consists of a square of side a with charges $\pm$ q at alternate corners. The square rotates with angular velocity ω about an axis normal to the plane of the square and through its center. Calculate in a long-wave length approximation:

a) The quadrupole moments

b) The radiation fields

c) The angular distribution of radiation.

Physics Qualifying Examination UCI September 28, 1977

THERMODYNAMICS & STATISTICAL MECHANICS

(3 hours)

Closed Book.

A. Thermodynamics -- do two of the three problems

1. Derive the following relations:

(a) $$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

where S is the entropy, V is the volume, p is the pressure and T is the absolute temperature

(b) $$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

where E is the internal energy.

(c) $$C_p - C_V = \frac{TV\alpha^2}{\kappa}$$

where C_p and C_V are the heat capacities at constant pressure and volume, respectively, α is the coefficient of volume expansion, $\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

and κ is the isothermal compressibility, $-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$.

2. Consider a gas whose equation of state is

$$p(V-b) = RT\left(1-\frac{a}{RTV}\right) .$$

(a) Prove that C_V is independent of the volume and hence depends only on the temperature.

(b) Derive an expression for the internal energy E which exhibits explicitly the dependence of E on V.

(c) Derive an expression for the entropy S which reveals explicitly the dependence of S on V.

3. Ten grams of a paramagnetic salt obeying Curie's law are in a magnetic field of 100,000 oersteds and at a temperature of $10^{\circ}K$. Assume that the heat capacity is constant at 0.10 cal/gm deg. and that the Curie constant is 0.05 deg./gm.

(a) If the field is reduced reversibly and isothermally to zero, calculate the heat transferred.

(b) If the field is reduced reversibly and adiabatically to zero, calculate the temperature change.

B. <u>Statistical Mechanics</u> -- do two of the three problems

1. The Fermi-Dirac distribution law for the probability f_k that a state with energy ϵ_k is occupied is

$$f_k = \frac{1}{e^{(\epsilon_k - \mu)/k_B T} + 1}$$

where μ is the Fermi energy. Consider a perfect Fermi gas of spin $\frac{1}{2}$ particles characterized by the energy

$$\epsilon_k = \frac{\hbar^2\left(k_x^2 + k_y^2 + k_z^2\right)}{2m}$$

and by the degeneracy parameter

$$y = \frac{nh^3}{2(2\pi m k_B T)^{3/2}}$$

where n is the number density, m is the mass, T is absolute temperature, and $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ is the wave vector.

(a) Taking the components of $\vec{k}$ to be specified by periodic boundary conditions applied to a large cubic container, derive a relation between y and μ. This relation should involve an integral over energy. Evaluate this relation explicitly in the limits of weak degeneracy and of strong degeneracy.

(b) Calculate the total kinetic energy and the pressure in the limit of strong degeneracy.

2. A one-dimensional Ising model with cyclic boundary conditions has the Hamiltonian

$$H = - J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} \quad , \qquad \sigma_{N+1} = \sigma_1$$

where J is the exchange constant for nearest neighbor interactions and the possible values of σ_i are ± 1.

(a) Calculate the partition function

(b) In the limit $N \to \infty$, calculate the specific heat at constant volume. Make a rough plot of specific heat versus absolute temperature.

(c) Is there a phase transition?

3. Consider a system of spins each of which has an energy of interaction with an effective magnetic field H_e given by

$$E = g\mu H_e S_z \quad .$$

The possible values of S_z are 0, ± 1, ± 2, ... $\pm S$.

(a) Show that the ensemble average of S_z is given by

$$\langle S_z \rangle = S B_S(\alpha)$$

where $B_S(\alpha)$ is the Brillouin function,

$B_S(\alpha) = \left(1 + \frac{1}{2S}\right) \coth \left(S + \frac{1}{2}\right)\alpha - \frac{1}{2S} \coth \frac{\alpha}{2}$, and

$\alpha = g\mu H_e / k_B T$.

(b) If the effective field H_e is given by

$$H_e = \frac{cJ}{g\mu} \langle S_z \rangle$$

where c is the number of nearest neighbors of a given spin and J is the exchange constant, show that a critical temperature T_c exists defined by $k_B T_c = \frac{1}{3} cJS(S+1)$.

(c) For temperatures just below the critical temperature, show that

$$\langle S_z \rangle \sim (T_c - T)^{\frac{1}{2}} .$$

Note: The following expansion may be useful:

$$\coth x = \frac{1}{x} + \frac{1}{3} x - \frac{1}{45} x^3 .$$

Physics Qualifying Examination September 28, 1977

QUANTUM MECHANICS II (Qualitative)

($1\frac{1}{2}$ hours)

Closed Book.

1. Consider the helium atom. Estimate the ground state energy either by using perturbation theory or by the variational method.

   Potentially useful integrals and formulas are:

$$\int d^3r \quad e^{-2\kappa r} = \frac{\pi}{\kappa^3}$$

$$\int d^3r \quad \frac{e^{-2\kappa r}}{r} = \frac{\pi}{\kappa^2}$$

$$\int d^3r_1 \, d^3r_2 \quad \frac{e^{-2\kappa(r_1+r_2)}}{|\vec{r}_1 - \vec{r}_2|} = \frac{5}{8} \frac{\pi^2}{\kappa^5}$$

$$\nabla_r^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r}\right)$$

2. Given the potential energy

$$V(x) = V_o(x-a)^2(x+a)^2$$

$$V_o = \frac{\kappa a^2}{8}$$

discuss and sketch the eigen-functions and associated energy eigenvalues for $E \ll V_o$ and $E \gg V_o$. Discuss the effect on adjacent energy levels as a becomes very large.

3. Molecules sometimes behave like rotators. If rotational spectra are characterized by radiation of wavelength of order 10^7 Å and this is used to estimate interatomic distances in a molecule like H_2, what kind of separations (in Å) are obtained?

4. The Hamiltonian for the Zeeman effect in the presence of spin orbit coupling is given approximately by

$$\hat{H} = \beta(\hat{\vec{L}} + 2\hat{\vec{S}}) \cdot \vec{B} + 2\alpha \hat{\vec{L}} \cdot \hat{\vec{S}}$$

where β is the Bohr magneton; $\hat{\vec{L}}$ and $\hat{\vec{S}}$ are orbital and spin angular momentum operators respectively and α is the spin-orbit coupling constant. Note that

$$[\hat{H}, \hat{L}^2] = [\hat{H}, \hat{S}^2] = 0$$

so that no mixing occurs between states of different (ℓ, s). Consider <u>one</u> electron in a p-state ($\ell=1, s=\frac{1}{2}$). Describe in words how you would find the energy levels of this electron in the weak field limit and the strong field limit. What basis functions would you use in each limit? Sketch the energy levels and label them with appropriate quantum numbers.

Physics Qualifying Exam UCI September 28, 1977

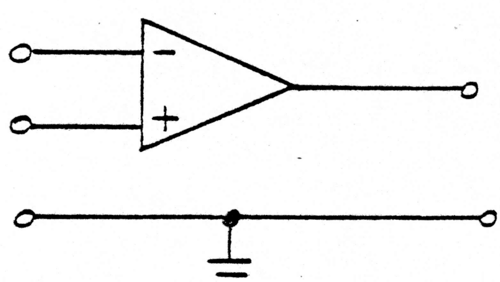
GENERAL PHYSICS

(Strive to do ten of the twelve questions. If you do ten, you are doing extremely well!)

1. Can a photon in vacuum decay into a system of n massive particles? Show your math.

2. Why will an ultra-cold niobium ball levitate in a magnetic field in defiance of gravity? Explain how, by applying a square wave voltage at the natural frequency of oscillation of the levitating ball, and utilizing a SQUID detector, one can look for quarks. Why would a niobium sphere be preferable to a hydrocarbon sphere for this purpose?

3. An ideal operational amplifier has infinite gain, infinite input impedance and zero output impedance. Its symbol is shown at the left. Show how to use an op amp to build:



a) An inverting variable gain amplifier. (Why is it stable?)
b) A non-inverting amplifier
c) A differential amplifier
d) A summation circuit
e) An integrator
f) A differentiator
g) Show how to combine four diodes and a reference oscillator to create a phase-sensitive detector (i.e. heart of a lock-in amplifier).

4. What is a Legendre transformation? Why is it relevant to
a) the definition of thermodynamic free energy,
b) the connection between Hamiltonian and Lagrangian mechanics.

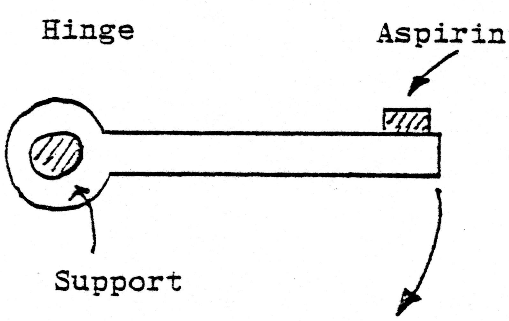
5. Consider a spherical universe of constant mass density ρ. Suppose it expands with a velocity $v(R)$ where

$$v(R) = H_R \quad ,$$

H being the Hubble constant and R the radius. Expansion will continue forever if the kinetic energy of the matter exceeds the potential energy. Find the value of ρ for which this is so, in terms of H.

6. The 1976 Nobel Prize in Physics was awarded to B. Richter of SLAC and S.C.C. Ting of MIT. Describe the nature of their discoveries.

7. A rigid rod pendulum, hinged at one end, is held in place in horizontal position with an aspirin resting on the end. When the rod is let go the end of the rod initially accelerates faster than the aspirin. Explain.



8. A polar semi-conductor such as InSb is doped so that it contains a certain concentration of free carriers. What is the qualitative nature of the optical absorption spectrum from the far infrared to the far ultraviolet?

9. Discuss how quarks make up baryons and mesons. What different kinds of quarks do you know of and what kinds of particles do they make up? What are the angular momenta of the s-wave baryon and meson states in the quark model?

10. A plasma in a fusion device has a number density of 10^{14} cm^{-3} of electrons and ions, each. Their temperature is 10 keV. Make some assumption, then compute the magnetic field necessary to contain the plasma.

11. The earth is slightly flattened at the poles and it bulges at the equator. The plane of the equator is inclined at an angle of about 25^{o} with respect to the plane of the earth's rotation about the sun. In one model of the ice ages this gives rise to periodic advance and retreat of glaciers from the pole, with a time scale of about 25,000 years. Without doing math, but exploiting the analogy to a well known problem in mechanics, explain the connection. (Hint: It is related to the fact that the Pole star changes from Polaris to Lyra and back on the same 25,000 year time scale.)

12. a) Why is the sky blue? Why is a sunset red?
   b) If a layer of liquid is heated from below what changes occur as the thermal gradient between bottom and top increases?
   c) List the four known fundamental forces of nature in order of increasing strength.

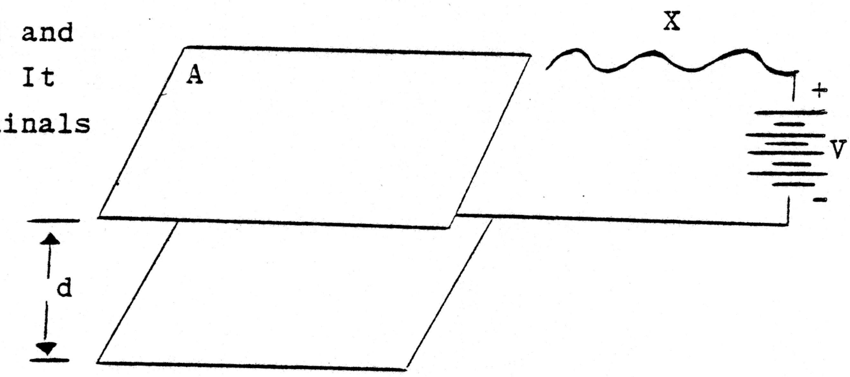
UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART I - Electricity and Magnetism

- Time: 0900 - 1200 = 3 hours, May 22
- Do 4 of the 6 problems
- The 4 problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

Part I - Electricity and Magnetism

1. A gas-filled parallel plate ionization chamber has plates of area A separated by d and a capacitance $C = A\varepsilon_o/d$. It is connected to the terminals of a battery as shown.



(a) What charge and energy are taken from the battery in the charging process? What charge and energy are stored in the capacitor? What is the electrostatic force on one plate?

(b) An ultra-violet photon creates an electron-positive ion pair midway between the plates at $t = 0$. Both the electron and positive ion undergo frequent gas atom collisions and so move parallel to the electric field with small constant drift velocities, $|v_e| \approx 1000|v_+|$. An instrument connected in series at X measures charge $q(t) = \int_o^t i(t)dt$. Find $q_e(t)$ and $q_+(t)$ that result from the motion of the electron and positive ion. Show results on a labelled sketch. Do not include image charges or space charge effects in your analysis.

2. In general, when one produces a beam of ions or electrons, the space charge within the beam causes a potential difference between the axis and the surface of the beam. A 10 m A beam of 50 keV protons ($v = 3 \times 10^6$ m/sec) travels along the axis of an evacuated beam pipe. The beam has a circular cross section of 1 cm diameter. Calculate the potential difference between the axis and the surface of the beam, assuming that the current density is uniform over the beam diameter.

3. Show that a charge, acted upon by static electric forces only, cannot be in stable equilibrium (this is called "Earnshaw's theorem"). Using this result, show that the maximum value of the absolute value of a function of a complex variable which is regular in a closed region always lies on the boundary of that region (this is called "the maximum modulus principle").

4. An infinite sheet of current, located in the xz-plane, has a current density J (amperes/m) given by

$$J = J_z = J_o e^{i\omega t}$$

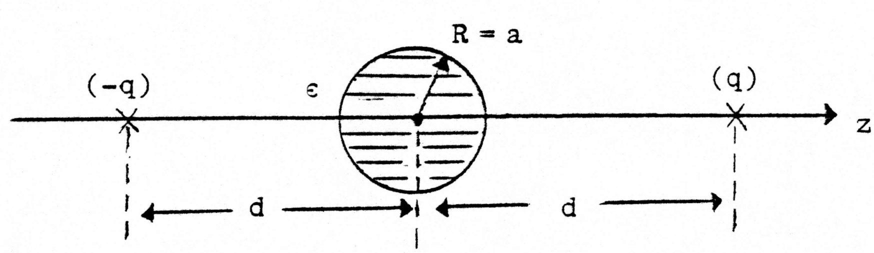
where J_o is constant over the x z plane.

(a) Use symmetry arguments to determine the directions of the electric field $\vec{E}$ and the magnetic field $\vec{B}$.

(b) Obtain $\vec{B}$ from the integral form of ampere's law, showing that the displacement-current term does not contribute. What is the wavelength of $\vec{B}(\vec{r},t)$?

(c) What is the power per unit area radiated from one side of the current sheet?

5. Consider two point charges of equal magnitude and opposite sign located at distances $z = \pm d$ from a dielectric sphere of radius a (dielectric constant ε) positioned at the origin



5. (continued)

(a) If $\varepsilon = 1$ find an expression for the potential everywhere in space (express potential in terms of Legendre polynomials, check value at the origin).

(b) If $\varepsilon > 1$, find expressions for the form of the potential inside and outside the dielectric sphere.

(c) Use appropriate boundary conditions to find the potential everywhere inside the sphere.

(d) Find the induced surface charge density, and verify that the total induced surface charge is zero.

6. Find the convection and displacement currents and the resulting magnetic field everywhere in space due to a slowly moving, uniformly dense, spherical cloud of charge. Let the cloud have a radius a, total charge e, and let it move along +z axis with velocity $v \ll c$.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART II - Quantum Mechanics I (Quantitative)

- Time: 1300 - 1730 = 4.5 hours, May 22
- Do all 6 problems
- The 6 problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART II - Quantum Mechanics I (Quantitative)

1. a) Consider a typical set of 2j + 1 angular momentum eigenstates $|\, jm \rangle$; as usual

$$\vec{J}^2 |jm\rangle = j(j+1)|jm\rangle$$

$$J_z = |jm\rangle = m|jm\rangle$$

($\vec{J}$ = angular momentum, and we set $\hbar = 1$)

Define the operator $T = \sum_{ijk} \varepsilon_{ijk} J_i J_j J_k = \vec{J} \cdot (\vec{J} \times \vec{J})$.

Are the states $|jm\rangle$ eigenstates of T? If so, give the eigenvalues.

If not, construct from the $|jm\rangle$ at least one eigenstate of T.

b) The Pauli spin matrices are a two-dimensional representation of the angular momentum operators J_x, J_y, J_z. Give the analogous three-dimensional representation of these operators.

2. Let the magnetic moment operator of a particle of mass m be $\vec{\mu} = -g \frac{e}{2mc} \vec{J}$ where $\vec{J}$ is its spin operator. The polarization $\vec{\sigma}$ of the particle is the expectation value of its spin, and let $\vec{p}$ be the expectation value of its momentum. If the particle is acted upon by a uniform magnetic field $\vec{B}$ perpendicular to $\vec{p}$ and $\vec{\sigma}$, and $\vec{p}$ is turned thru and angle θ_p, find the angle θ_σ thru which $\vec{\sigma}$ is turned.

3. A particle of mass m is constrained to move on the surface of the sphere $r = a$, but is otherwise free.

a) What is the energy difference between the ground state and the first excited state of this system?

Gravity is now turned on, i.e. the potential energy $V = mgz = mga\cos\theta$ is added to the Hamiltonian.

b) The pattern of energy levels is completely different in the "weak gravity" limit $g \ll g_o$ and the "strong gravity" limit $g \gg g_o$. Evaluate the critical value g_o, omitting dimensionless constants of order unity.

c) What is the energy difference between the ground state and the first excited state in the "strong gravity" limit?

4. a) A particle of mass m moves in the Yukawa potential $V(r) = -\lambda \frac{e^{-\mu r}}{r}$.
Use the variational method, with a trial wave function $\Psi \sim e^{-\alpha r}$, to estimate the critical value λ_o of the parameter λ; λ_o is defined by

$$\lambda < \lambda_o \rightarrow \text{no bound states present}$$

$$\lambda > \lambda_o \rightarrow \text{at least one bound state present}$$

b) Suggest simple trial functions to represent the 1p and 2s states and indicate briefly the reasoning behind your choice.

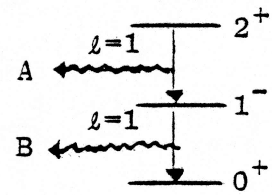
5. The electrostatic potential of a neutral atom of atomic number Z is to a good approximation a screened Coulomb potential

$$V(r) = \frac{Ze}{r} e^{-(r/a)}$$

where the exponential represents the shielding of the nuclear charge by the atomic electrons. The shielding length a, according to the Thomas-Fermi theory of the atom, is approximately $a_o/Z^{1/3}$, where a_o is the hydrogen Bohr radius.

Find the total scattering cross-section, using the Born approximation, for non-relativistic electrons scattering from such an atom.

6. A nucleus is initially in an unpolarized 2^+ state. It decays to a 1^- state by emitting a p-wave spinless particle A, and this state immediately decays to the 0^+ ground state by emitting another p-wave spinless particle B, as sketched below.



Note that a table of Clebsch-Gordan coefficients is given on the next page.

a) If the axis of polar coordinates is chosen along the direction of emission of particle A, what is the probability that the intermediate 1^- state has m $(=J_z/h)$ equal to +1? to zero? to -1?

b) What is the angular distribution $W(\theta)$ (i.e., the so-called angular correlation) of particle B referred to this same coordinate system?

c) Explain briefly the significance of the requirement in part a) that the 1^- state decay "immediately". For example, what is likely to happen in practical cases, if the intermediate state has too long a lifetime?

CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...
m_1 m_2		
m_1 m_2	Coefficients	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2 × 1/2

m_1	m_2	J=1	J=0
M = +1			
+1/2	+1/2	1	
M = 0			
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2
M = -1			
-1/2	-1/2	1	

1 × 1/2

m_1	m_2	J=3/2	J=1/2
M = +3/2			
+1	+1/2	1	
M = +1/2			
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3
M = -1/2			
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3
M = -3/2			
-1	-1/2	1	

2 × 1/2

m_1	m_2	J=5/2	J=3/2
M = +5/2			
+2	+1/2	1	
M = +3/2			
+2	-1/2	1/5	4/5
+1	+1/2	4/5	-1/5
M = +1/2			
+1	-1/2	2/5	3/5
0	+1/2	3/5	-2/5
M = -1/2			
0	-1/2	3/5	2/5
-1	+1/2	2/5	-3/5
M = -3/2			
-1	-1/2	4/5	1/5
-2	+1/2	1/5	-4/5
M = -5/2			
-2	-1/2	1	

3/2 × 1/2

m_1	m_2	J=2	J=1
M = +2			
+3/2	+1/2	1	
M = +1			
+3/2	-1/2	1/4	3/4
+1/2	+1/2	3/4	-1/4
M = 0			
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2
M = -1			
-1/2	-1/2	3/4	1/4
-3/2	+1/2	1/4	-3/4
M = -2			
-3/2	-1/2	1	

2 × 1

m_1	m_2	J=3	J=2	J=1
M = +3				
+2	+1	1		
M = +2				
+2	0	1/3	2/3	
+1	+1	2/3	-1/3	
M = +1				
+2	-1	1/15	1/3	3/5
+1	0	8/15	1/6	-3/10
0	+1	6/15	-1/2	1/10
M = 0				
+1	-1	1/5	1/2	3/10
0	0	3/5	0	-2/5
-1	+1	1/5	-1/2	3/10
M = -1				
0	-1	6/15	1/2	1/10
-1	0	8/15	-1/6	-3/10
-2	+1	1/15	-1/3	3/5
M = -2				
-1	-1	2/3	1/3	
-2	0	1/3	-2/3	
M = -3				
-2	-1	1		

1 × 1

m_1	m_2	J=2	J=1	J=0
M = +2				
+1	+1	1		
M = +1				
+1	0	1/2	1/2	
0	+1	1/2	-1/2	
M = 0				
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3
M = -1				
0	-1	1/2	1/2	
-1	0	1/2	-1/2	
M = -2				
-1	-1	1		

3/2 × 1

m_1	m_2	J=5/2	J=3/2	J=1/2
M = +5/2				
+3/2	+1	1		
M = +3/2				
+3/2	0	2/5	3/5	
+1/2	+1	3/5	-2/5	
M = +1/2				
+3/2	-1	1/10	2/5	1/2
+1/2	0	3/5	1/15	-1/3
-1/2	+1	3/10	-8/15	1/6
M = -1/2				
+1/2	-1	3/10	8/15	1/6
-1/2	0	3/5	-1/15	-1/3
-3/2	+1	1/10	-2/5	1/2
M = -3/2				
-1/2	-1	3/5	2/5	
-3/2	0	2/5	-3/5	
M = -5/2				
-3/2	-1	1		

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART III - Mathematical Physics

- Time: 9:00 - 12:00 (3 hours), May 23
- Do 4 of the 5 problems
- The problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART III - Mathematical Physics

1. a) Consider the ordinary differential equation:

$$xy'' - (1 + x)y' + 2(1 - x)y = 0$$

i) Find the first three terms in a power series expansion of two independent solutions about the origin.

ii) Do these solutions have singularities at any finite point of the complex x-plane?

b) i) Define the "characteristic curves" of a second order linear partial differential equation in two variables. Explain the classification of the latter into hyperbolic, parabolic and elliptic equations.

ii) What sort of boundary curve and what boundary conditions thereon, are suitable for the equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + 1000 \frac{\partial^2 \Psi}{\partial y^2} = 27 \frac{\partial \Psi}{\partial y}$$

iii) If Ψ and $\frac{\partial \Psi}{\partial y}$ are specified in $0 \leq x \leq 1$ for the equation:

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = 0$$

over what (if any) range of x and y, can Ψ be determined from the equation?

2. Consider the integral equation

$$f(x) = g(x) + \lambda \int_a^b K(x,x') f(x') dx'$$

2. (continued)

written in operator form as

$$|f\rangle = |g\rangle + \lambda\, K\, |f\rangle$$

i) Write out the Neumann series for this equation.

ii) What is the qualitative form of the Fredholm solution and what advantage does this solution have over the Neumann series?

iii) For what sort of kernel K would you use the Wiener-Hopf method?

b) Find eigenfunctions and eigenvalues λ of

$$f(x) = \lambda \int_{-1}^{+1} \left[1 + \sqrt{\tfrac{3}{2}}\,(x+y) + 6\,xy\right] f(y)\,dy$$

3. Find the first two terms in the asymptotic expansion for large real ν of

$$F(\nu) = \int_0^1 \exp\,[i\nu x^2]\,dx$$

4. a) i) Find the Fourier transform of the polynomial

$$P_n(x) = \sum_{m=0}^{n} a_m x^m$$

ii) Give a rigorous definition, within the context of the theory of distributions (generalized functions), of the "functions" which occur in the solution of (i).

b) Define

$$\phi(x,y) = \sum_{m=-\infty}^{\infty} \exp\left[-\pi(m+y)^2 x\right]$$

for $x>0$, and one particular theta function by

$$\theta(x) = \phi(x,0).$$

Find the Fourier series expansion of $\phi(x,y)$ of the form

$\sum\limits_{n=-\infty}^{\infty} a_n(x)e^{2\pi iny}$ and hence derive a functional relation between $\theta(x)$ and $\theta\left(\frac{1}{x}\right)$.

5. i) A coin is tossed n times with probability x of heads and 1-x of tails. Find the probability that heads turns up exactly k times in the n independent trials.

ii) Let y_i be independent random variables taking the values 0 and 1 with probability 1-x and x respectively. Define

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} y_i$$

a) Write down the probability distribution of Z_n.

Now define

$$B_n(f,x) = \langle f(Z_n) \rangle \text{ where}$$

f(x) is any continuous function of x in [0,1] and < > denotes expectation (or mean) value.

b) Derive an expression for $B_n(f,x)$ in terms of $f\left(\frac{k}{n}\right)$ (k is an integer between 0 and n) and show that, for fixed f, $B_n(f,x)$ is a polynomial in x of degree n.

c) Use the central limit theorem to find a simple form for $B_n(f,x)$ as $n \to \infty$.

d) Indicate, briefly and without being rigorous, how the previous results can be used to show that any continuous function can be uniformly approximated by polynomials. (Weierstrass's theorem).

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART IV - Classical Mechanics

- Time: 1:00 - 5:30 PM, (3.5 hours), May 23
- Do 5 of the 6 problems
- The problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

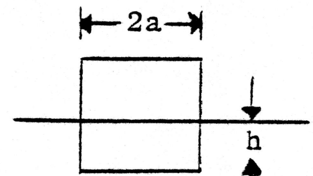
PART IV - Classical Mechanics

1. Estimate numerical values for the following quantities, and indicate how you arrive at your estimate.
   a) The speed, in m/sec, of a comet at perigee = 0.1 A.U.
   b) The magnetic field, in gauss, near the ground underneath a typical power transmission line feeding the Los Angeles area, due to the line only.
   c) The speed, expressed as a fraction of the velocity of light, of a 200 GeV/c proton.
   d) The pressure, in Newton's/m^2, of a 100 mi/hr wind blowing on the side of a building.
   e) The amount of energy, in joules, required to raise an isolated metal sphere of 1 meter radius to a million volts.

2. Three masses m_1, m_2 and m_3 move subject to their mutual gravitational attraction. If the masses are initially at the vertices of an equilateral triangle of side a, and if their initial velocities are suitably chosen, their locations will continue to define a (rotating) equilateral triangle of side a. Express ω, the angular frequency of rotation of the triangular configuration about its center of mass, as a function of the masses, of the side a, and of the gravitational constant G. (It is interesting to note that several asteroids, called Trojan asteroids, appear to form such a configuration with Jupiter and the Sun.)

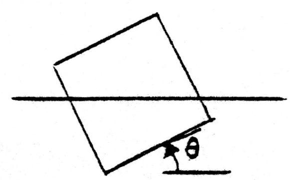
3. A long beam with a square cross-section (side = 2a) floats on water. The beam has density s; the water has density 1.

a) The beam is in equilibrium with one side of the square parallel to the water surface, as shown below.



Calculate h, the depth of immersion.

b) For certain densities s, there is a second equilibrium position, in which a trapezoidal portion of the cross-section is submerged, as shown below.



Calculate the tangent of the inclination θ as a function of the density s.

c) What is the range of densities s for which such a tilted equilibrium position, with a <u>trapezoidal</u> submerged portion, exists? Remark: In this problem we do not distinguish between stable and unstable equilibrium; both are referred to as equilibrium.

4. A system undergoes small oscillations about equilibrium; expressed in terms of normal coordinates $q_k(t)$, the kinetic and potential energies are

$$T = (1/2)\sum_{k=1}^{n} \dot{q}_k^2 \qquad V = (1/2)\sum_{k=1}^{n} \lambda_k q_k^2$$

4. (continued)

a) What are the normal frequencies of oscillation?

A constraint is now imposed on the system, namely $\sum_{k=1}^{n} A_k q_k = 0$.
The kinetic and potential energies are unaffected.

b) Write the explicit equation whose roots now give the new frequencies of oscillation.

Show from this equation that the new frequencies interleave the old frequencies; i.e., if the old frequencies are $\omega_1 > \omega_2 > \ldots > \omega_n$, and the new frequencies are $\Omega_1 > \Omega_2 > \ldots > \Omega_{n-1}$, then show that $\omega_1 > \Omega_1 > \omega_2 > \Omega_2 > \ldots > \Omega_{n-1} > \omega_n$.

5. A circular metal plate of radius a is rigidly clamped around its perimeter. The differential equation describing small oscillations of the plate is

$$\nabla^4 u + \alpha \frac{\partial^2 u}{\partial t^2} = 0$$

where $\nabla^4 u = \nabla^2 (\nabla^2 u)$ and α is a constant involving the elastic properties of the metal. The boundary conditions at $r = a$ are

$$u = 0 \text{ and } \frac{\partial u}{\partial r} = 0$$

(i) Consider the equivalent mathematical problem of solving

$$\nabla^4 u - k^4 u = 0$$

inside the unit circle, with $u = 0$, $\frac{\partial u}{\partial r} = 0$ at $r = 1$.
Let the smallest eigenvalue of this equation be k_o^4. What is the frequency, ω_o, of the lowest mode of our plate in terms of k_o, a, and α?

5. (continued)

(ii) By noting the factorization $\nabla^4 - k^4 = (\nabla^2 - k^2)(\nabla^2 + k^2)$ give two linearly independent solutions of $\nabla^4 u - k^4 u = 0$ which are appropriate to the problem.

(iii) By imposing the boundary conditions of (i), find a transcendental equation whose roots are the eigenvalues k.

(iv) If the perimeter of the plate is free rather than rigidly clamped, what are the boundary conditions?

6. Consider the Lagrangian

$$L = m\, e^{\gamma t}\, \frac{1}{2} (\dot{x}^2 - \omega^2 x^2)$$

for the motion of a particle of mass m in one dimension (x). The constants m, γ and ω are real and positive.

(a) Find the equation of motion

(b) Interpret the equation of motion by stating the kinds of forces to which the particle is subject.

(c) Find the canonical momentum, and from this construct the Hamiltonian function.

(d) Is the Hamiltonian a constant of the motion? Is the energy conserved? Explain.

(e) For the initial condition $x(0) = 0$ and $\dot{x}(0) = + v_o$, what is x(t) asymptotically as $t \to \infty$?

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

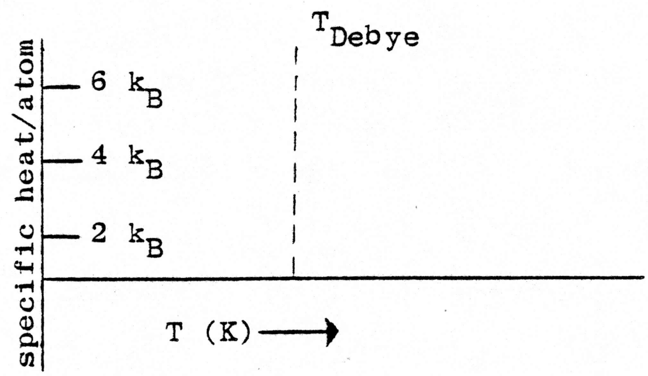
PART V - Thermodynamics and Statistical Mechanics

- Time: 0900 - 1200 = 3 hours, May 24
- Do all 4 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART V - Thermodynamics and Statistical Mechanics

1. Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photo-conducting resistance. Diamond is a monoatomic dielectric solid of carbon having 10^{21} atoms cm^{-3}; on the graph below, sketch in, roughly, its specific heat (per atom) as a function of absolute temperature.

a)



b) How is T_{Debye} related to the Debye frequency, ω_D?

c) If the acoustic velocity at low frequencies is 5×10^5 cm/sec, approximately what is the value of ω_D?

2. A material is brought from temperature T_i to temperature T_f by placing it in contact with a series of N reservoirs at temperatures $T_i + \Delta T$, $T_i + 2\Delta T, \ldots, T_i + N\Delta T = T_f$. Assuming that the heat capacity of the material, C, is temperature independent, calculate the entropy change of the total system, material plus reservoirs. What is the entropy change in the limit $N \to \infty$ for fixed $T_f - T_i$?

3. The energy levels of a three-dimensional rigid rotor of moment of inertia I are given by

$$E_{J,M} = \frac{\hbar^2}{2I} J(J+1)$$

where $J = 0, 1, 2, \ldots; M = -J, -J+1, \ldots, J$. Consider a system of N rotors:

a) Using Boltzmann statistics, find an expression for the thermodynamical internal energy of the system.

b) Under what conditions can the sums in part (a) be approximated by integrals? In this case calculate the specific heat C_v of the system.

4. A dilute gas of atoms having $S = 1$, $L = 1$ and $J = 1$ is located in a uniform magnetic field, B. Using quantum mechanics and a assuming that $\mu_{Bohr} B \ll k T$, find:

a) The energy levels for the three spin projections along the field direction.

b) The contribution to the specific heat due to the magnetic moment.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART VI - Quantum Mechanics II (Qualitative)

- Time: 1300-1430 = 1.5 hours, May 24
- Do all 4 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART VI - Quantum Mechanics II (Qualitative)

1. Suppose an electron is in a state described by the wave function

$$\psi = \frac{1}{\sqrt{4\pi}} (e^{i\varphi}\sin\theta + \cos\theta)\, g(r)$$

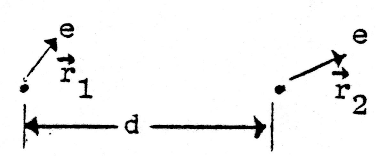
where $\int_0^\infty |g(r)|^2 r^2\, dr = 1$

and φ, θ are the azimuth and polar angles, respectively.

(a) What are the possible results of a measurement of the z-component of the angular momentum L_z of the electron in this state?

(b) What is the probability of obtaining each of the possible results in part (a)?

(c) What is the expectation value of L_z?

2. 

Consider the following model for the Van der Waals force between two atoms: Each atom consists of one electron bound to a very massive nucleus by a potential $V(r_i) = \frac{1}{2} m \omega^2 r_i^2$. Assume that the two nuclei are d ($d \gg \sqrt{\hbar/m\omega}$) apart along the x-axis and that there is an interaction $V_{12} = \beta \frac{x_1 x_2 e^2}{d^3}$ (Ignore the fact that the particles are indistinguishable).

(a) Consider the ground state of the entire system when $\beta=0$. Give its energy and wave function in terms of $\vec{r}_1$ and $\vec{r}_2$.

(b) Calculate the lowest non-zero correction to the energy (ΔE) and to the wave function due to V_{12}.

(c) Calculate the r.m.s. separation along the x direction of the two electrons, to lowest order in β.

2. (continued)

$$\psi_o(x) = \langle x|0\rangle = \left(\sqrt{\frac{m\omega}{\pi\hbar}}\right)^{\frac{1}{2}} e^{\frac{-x^2 m\omega}{2\hbar}} \;;\; \psi_1(x) = \langle x|1\rangle =$$

$$= \left(\sqrt{\frac{m\omega}{\pi\hbar}}\ \frac{2m\omega}{\hbar}\right)^{\frac{1}{2}} x\, e^{\frac{-x^2 m\omega}{2\hbar}} \;;$$

$$\langle n|x|m\rangle = 0 \text{ for } |n-m| \neq 1 \;,\quad \langle n-1|x|n\rangle = (n\,\hbar/2m\,\omega)^{\frac{1}{2}} ,$$

$$\langle n+1|x|n\rangle = ([n+1]\,\hbar/2m\,\omega)^{\frac{1}{2}}.$$

3. (a) Suppose the state of a certain harmonic oscillator with angular frequency ω is given by the wave function

$$\psi = N \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, \psi_n(x)\, e^{-in\omega t},\quad \alpha = x_o\sqrt{\frac{m\omega}{2\hbar}}\, e^{i\varphi},\quad N = e^{-\frac{1}{2}|\alpha|^2} .$$

Calculate the average position of the oscillator $\langle x\rangle$ in this state, and show that the time-dependence of $\langle x\rangle$ is that of a classical oscillator with amplitude x_o and phase φ.

(b) The Hamiltonian for a one-dimensional harmonic oscillator in a laser electromagnetic field is given by

$$H = \frac{p^2}{2m} + \frac{ep}{2m\omega} E_o \sin \omega t - \tfrac{1}{2}\, e\, E_o x \cos \omega t + \tfrac{1}{2}\, m\, \omega_o^2 x^2 .$$

Where ω_o, m, and e are the angular frequency, mass, and charge of the oscillator, and ω is the angular frequency of the radiation. Assume the laser is turned on at t = 0 with the oscillator in its ground state ψ_o. Treat the electromagnetic interaction as a perturbation, in first order, and find the probability for any time t > 0 that the oscillator will be found in one of its excited states ψ_n.

Useful information: The normalized oscillator wavefunctions $\psi_n(x)$ have the property that

$$\left(x + \frac{\hbar}{m\omega}\frac{d}{dx}\right)\psi_n = \sqrt{\frac{2\hbar}{m\omega}\cdot n}\ \psi_{n-1},\quad \left(x - \frac{\hbar}{m\omega}\frac{d}{dx}\right)\psi_n = \sqrt{\frac{2\hbar}{m\omega}\cdot(n+1)}\ \psi_{n+1}$$

4. The independent-particle shell model of the nucleus consists of non-interacting neutrons and protons moving in a three-dimensional harmonic oscillator potential.

a) Draw an energy level diagram for the 5 lowest states of this extreme shell model. Indicate degeneracies.

If we now consider the presence of a strong, central spin-orbit potential:

b) For ^{18}Ne (10 protons) list the spins, parities and isotopic spins for all the states in the $(1\ d_{5/2})^2$, $(1\ d_{5/2},\ 2\ s_{1/2})$ and $(2\ s_{1/2})^2$ configurations.

c) Predict for both the ground state and the first excited state of ^{17}F (9 protons) the spins, parities, and magnetic moments.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART VII - General Physics - The last part!

- Time: 1500 - 1800 = 3 hours, May 24
- Do 10 of the 13 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART VII - General Physics

1. Answer each of the following with, at most, a sentence.

   a) Give the principal decay mode of each:

   $$\mu^+,\ \pi^\circ,\ e^+,\ {}^{14}C,\ {}^{238}U.$$

   b) Give the spin and parity of each:

   $$\mu^+,\ \pi^+,\ {}^{2}H,\ {}^{12}C,\ \Delta^{++}.$$

   c) Give the approximate magnitudes of the:

   1) Proton energy at the center of the sun.
   2) Wavelength of a VHF TV signal.
   3) Experimental upper limit on the mass of the $\bar{\nu}_e$.
   4) Temperature at which ^{3}He exhibits superfluid properties.
   5) Critical magnetic field for superconducting lead at 0°K.

   d) What would you usually measure with:

   1) A Ge(Li) detector?
   2) An electron micro-probe?
   3) A Pitot tube?
   4) An electroscope?
   5) A Pirani gauge?

---

2. a) Describe briefly an experiment that would determine whether the ν_μ is different from the ν_e (or the $\bar{\nu}_\mu$ different from the $\bar{\nu}_e$).

   b) Assuming that the experiment described in a) establishes that the ν_μ and ν_e are different, then there must be a quantum number that distinguishes muonic leptons from electronic leptons. Assign

the lepton number as in the table below.

Lepton number	e^-	e^+	ν_e	$\bar{\nu}_e$	μ^-	μ^+	ν_μ	$\bar{\nu}_\mu$
L_e	1	-1	1	-1	0	0	0	0
L_μ	0	0	0	0	1	-1	1	-1

Describe an experiment that would distinguish which of the following two conservation laws is correct for a weak decay involving both muonic and electronic leptons:

a) $\sum_i L_e^{(i)}$ and $\sum_i L_\mu^{(i)}$ conserved separately.

("additive" conservation law)

b) $\sum_i (L_e^{(i)} + L_\mu^{(i)})$ and $\pi_i (-1)^{L_\mu(i)}$ conserved

("multiplicative" conservation law)

3. SLAC has used the following scheme for producing high-energy, monochromatic photons:

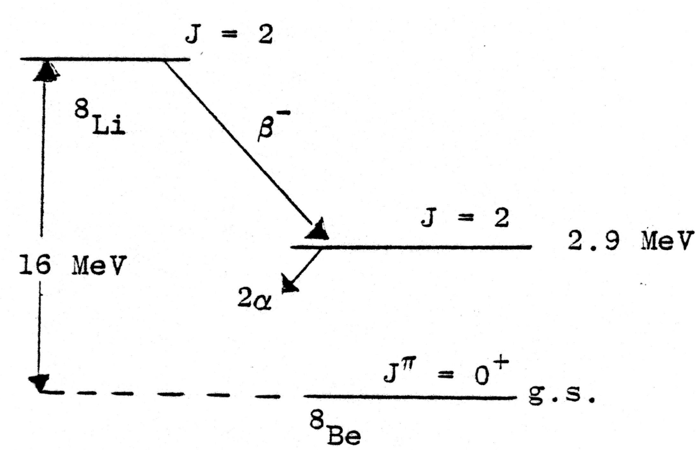
A beam of low-energy photons from a laser (visible light) is directed head-on into the 20 GeV electron beam. Find the energy of the scattered photons as a function of the angle they make with the electron beam and give the approximate value in GeV of the maximum photon energy.

4. A proposed experiment to measure the charge distribution of the pion involves the formation of "exotic atoms" μ^+-π^- or μ^--π^+.

a) What is the ground state energy in eV for the μ - π system if both are treated as point particles?

b) Find the energy correction in eV for the ground state of the "atom" assuming that the pion is a uniformly charged sphere whose radius is 10^{-13} cm.

5. ^{8}Li is an example of a β-delayed particle emitter. The ^{8}Li ground state has a half-life of 0.85 s and decays to the 2.9 MeV level in ^{8}Be. The 2.9 MeV level then decays into 2 alpha particles with a half-life of 10^{-22} s.



a) What is the parity of the 2.9 MeV level in ^{8}Be? Give your reasoning.

b) Why is the half-life of the ^{8}Be 2.9 MeV level so much smaller than the half-life of the ^{8}Li ground state?

c) Where in energy, with respect to the ^{8}Be ground state, would you expect the threshold for ^{7}Li neutron capture? Why?

6. a) Assuming that the two protons of the H_2^+ molecule are fixed at their normal separation of 1.06 Å, sketch the potential energy of the electron along the axis passing through the protons

b) Sketch the electron wave functions for the two lowest states in H_2^+, indicating roughly how they are related to hydrogenic wave functions. Which wave function corresponds to the ground state of H_2^+ and why?

c) What happens to the two lowest energy levels of H_2^+ in the limit that the protons are moved far apart?

7. The atomic number of Mg is Z = 12.

a) Draw a Mg atomic energy level diagram (not necessarily to scale) illustrating its main features, including the ground state and excited states arising from the configurations in which one valence electron is in the 3s state and the other valence electron is in the state $n\ell$ for $n = 3, 4$ and $\ell = 0, 1$. Label the levels with conventional spectroscopic notation. Assume LS coupling.

b) On your diagram, indicate the following (give your reasoning!):

i) an allowed transition

ii) a forbidden transition

iii) an intercombination line (if any)

iv) a level which shows (1) anomalous and (2) normal Zeeman effect, if any.

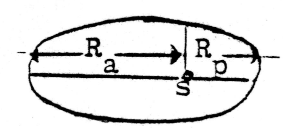
8. Find the threshold energy (kinetic energy) for a proton beam to produce the reaction

$$p + p \rightarrow \pi^\circ + p + p$$

with a stationary proton target.

9. An excellent camera lens of 60 mm focal length is accurately focussed for objects at 15 m. For what aperture (stop opening) will the diffraction blur of visible light be roughly the same as the defocus blur for a star (at ∞)?

10. A comet in an orbit about the sun has a velocity of 10 km/sec at apheliom and 80 km/sec at perihelion. If the earth's velocity in a circular orbit is 30 km/sec, and radius of its orbit is 1.5×10^8 km, find the aphelion distance R_a for the comet.



11. A particle X has two decay modes with partial decay rates $\gamma_1(\text{sec}^{-1})$ and $\gamma_2(\text{sec}^{-1})$.

a) What is the inherent uncertainty in the mass of X?

b) One of the decay modes of X is the strong interaction decay:

$$X \rightarrow \pi^+ + \pi^+$$

What can you conclude about the isotopic spin of X?

12. A non-relativistic electron of mass m, charge $-e$, in a cylindrical magnetron moves between a wire of radius a at a negative electric potential $-\varphi_o$ and a concentric cylindrical conductor of radius R at zero potential. There is a uniform constant magnetic field B parallel to the axis. Use cylindrical coordinates r, θ, z. The electric and magnetic vector potentials can be written

$$\varphi = -\varphi_o \frac{\ell n\, ^r/R}{\ell n\, ^a/R}\ , \quad \vec{A} = \frac{1}{2} Br\, \hat{\theta}$$ ($\hat{\theta}$ a unit vector in direction of increasing θ).

a) Write the Lagrangian and Hamiltonian functions.

b) Show that there are three constants of the motion, write them down, and discuss the kinds of motion which can occur.

c) Assuming that an electron leaves the inner wire with zero initial velocity, there is a value of the magnetic field B_c such that for $B \le B_c$ the electron can reach the outer cylinder, and for $B > B_c$ the electron cannot reach the outer cylinder. Find B_c and make a sketch of the electron's trajectory for this case. You may assume that $R \gg a$.

13. Give short answers including one sentence of reasons for the following ten questions:

a) How old is the universe?

b) What is Hubble's constant?

c) When was the most part of the Helium in the universe made?

d) When were the heavy elements (of which we are made) formed?

e) Is the universe closed?

f) What was the temperature of the universe when the black body radiation decoupled?

g) What is the baryon density of the universe?

h) How many photons are there per baryon?

i) How many neutrinos are there per baryon?

j) How big *was* the big bang?

"The big bang? Believe me, it was very, very, very, very, very, *very* big."

MATHEMATICAL PHYSICS
Open Book
Do all 3 problems

1. An infinite cylinder of radius a is heated. the temperature obeys

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{k}\frac{dT}{dt} .$$

The surface temperature is prescribed,

$$T(a,t) = f(t). \qquad t \geq 0$$

and initially, $T(r,o) = 0$.

(a) Find an expression for the temperature at all later times.

(b) If $f(t) = T_o \frac{t}{\tau}$, find $T(t)$.

For long times, what is the t-dependence?

2. A differential equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + 2 = 0$$

comes from minimizing some integral, I, over an area (x,y).

(a) What is the integrand of I.

(b) Now suppose we wish to find a function ψ which satisfies the condition that I be a minimum in the region and ψ be zero on the elliptical boundary

$$g = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 .$$

Try a choice $\psi = d_1 g$ and find d_1.

3. The random variable x has the probability distribution

$$f(x) = \frac{1}{\sqrt{2}} e^{-x^2/2} \qquad (-\infty < x < \infty)$$

(a) Find $\langle x \rangle$ and $\langle x^2 \rangle$.

(b) n independent measurements x_i are made. Let $\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$. Find $\varphi(\bar{x})$, the probability distribution of $\bar{x}$. Find $\langle \bar{x} \rangle$.

(c) Let $s = \frac{1}{n}(x_1^2 + x_2^2 + \ldots + x_n^2)$. Find $\psi(s)$, the probability distribution of s. Find $\langle s \rangle$.

Physics Qualifying Exam UCI Oct. 3, 1978

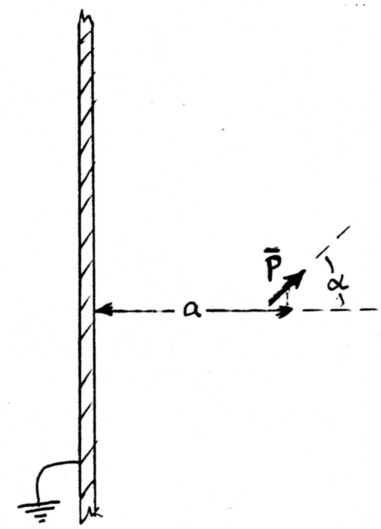
ELECTRICITY AND MAGNETISM

<u>Open Book</u>

(35) 1. For the dipole and infinite conducting plane shown, calculate:

i) the surface charge on the conductor; and

ii) the resultant torque on the dipole.



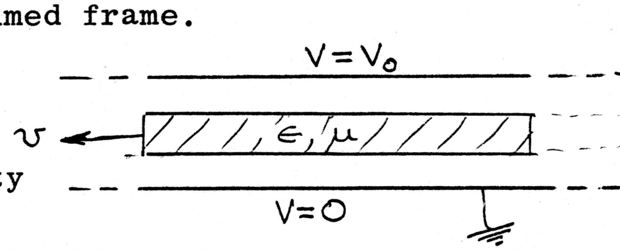
(20) 2. i) Use the Lorentz transformation of electric field and magnetic induction, and the result

$$\bar{P}'_{\perp} = \gamma(\bar{P}_{\perp} - \bar{v} \times \bar{M}_{\perp}/c^2)$$

$$\bar{M}'_{\perp} = \gamma(\bar{M}_{\perp} - \bar{v} \times \bar{P}_{\perp})$$

to derive the (perpendicular components of the) constitutive relations in the primed frame, given those in a medium in the unprimed frame.

ii) In the figure at the right, a slab of material with permeability $\underline{\mu}$ and permittivity $\underline{\epsilon}$ is moving with velocity $\underline{v}$ as shown. Give an expression, to first order in $\underline{v}$, for the current density $\underset{\sim}{J}$ (the source term in Ampere's law) in the lab frame. Do not carry out the details of the evaluation.



Sketch where the current would flow.

(45) 3. Given a uniform slab of plasma, occupying the volume $|z| \leq a$. Take as a fact that its dielectric constant can be expressed as

$$\epsilon = \epsilon_o \left(1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2}\right) .$$

where ω is an applied frequency and $\omega_{p\alpha}$ is the plasma frequency of species α.

Derive and plot the $(\omega - k)$ dispersion relation(s) for symmetric, slow ($\omega \ll kc$), small-amplitude waves, propagating as $\exp[i\omega t - ikx]$.

Discuss the z-profiles of the allowed excitations.

Note: Show that you can derive the wave electric field from a potential.

GENERAL PHYSICS

One Hour

Do all the problems

Open Book

(Points)

(5) 1. If a cup of tea is stirred with a rotary motion, the tea leaves are found to collect in the center of the bottom of the cup. Explain.

(15) 2. Estimate the total <u>classical</u> free energy of the present Sun ($M_\odot \sim 10^{30.3}$ kg, $R_\odot \sim 10^{8.8}$ m, $\langle T_\odot \rangle \sim 10^{7.1}$ ^{o}K), and thence its lifetime if there are no other energy sources.

(5) 3. The bulbs connected as shown are common 110 volt bulbs. When the circuit is closed, one bulb does not emit visible light. Which is it?

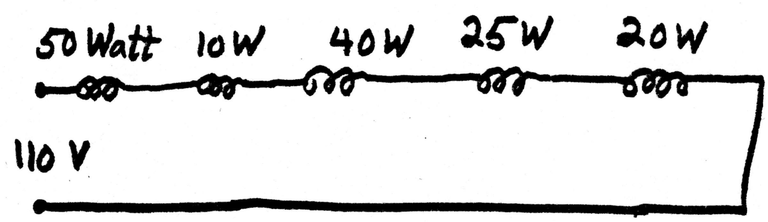
(a) 10 watt

(b) 20 watt

(3) 25 watt

(4) 40 watt

(5) 50 watt



(5) 4. An experimenter notices that the mercury column height of barometer 1, kept in the basement stands consistently <u>lower</u> than that of barometer 2, kept on the second floor. He discovers that the mercury of barometer 1 is contaminated with a little water. The best explanation of the difference in mercury heights is that:

a) The atmospheric pressure decreases with increasing altitude.

b) The weight of a given mass of mercury increases as the mercury is brought nearer to the earth.

c) Water vapor exerts a pressure.

d) Mercury evaporates less easily in the presence of water vapor.

e) The evaporation of water cools the mercury and causes it to contract.

(Points)

(5) 5. A scientist, while conducting an experiment, burns his finger. He finds it more comfortable to stick his finger in water rather than to wave it in the air. This indicates that:

a) Water is a better conductor than air, and the heat loss is mainly by conduction.

b) Water sets up stronger convection currents than air, and the loss is mainly by convection.

c) Water has a higher specific heat than air, and hence can absorb more heat.

d) A body radiates faster when surrounded by water than air, and the heat loss is mainly by radiation.

(5) 6. Which of the following would be MOST apt to undergo the LEAST temperature change over a period of a few hours?

a) A glass of water containing melting ice.

b) A pan of water covered with a tight-fitting lid and placed on a heating unit.

c) An open container of gasoline with air of constant temperature bubbling through it.

d) A melting ice cube placed in the freezing unit of a fast-freeze food locker.

e) The water at the bottom of a small pond while the surface is freezing over.

(10) 7. A bar magnet is supported by a delicate coiled spring attached to one end. The magnet is set into harmonic motion in a vertical line. Its period is T. Now there is brought up from below the lower end of the magnet a closed coil of wire so that the motion of the magnet is into and out of the coil. The following facts are all true, <u>except</u>:

a) The period of the motion will now be longer

b) The frequency of the motion will not be less

c) The motion of the magnet will not be altered by the presence of the coil

d) Lenz's Law gives rise to damping forces

e) an e.m.f. is induced in the coil and the coil acquires polarity

(Points)

8. A parallel plate condenser has two vertical plates 12 mm apart and a plate of glass 10 mm thick between the plates. The condenser (capacitor) is charged with +Q on one plate,

(5) -Q on the other, and then the charging connectors are removed. If now the glass plate is withdrawn:

a) mechanical work must be expended upon it

b) mechanical work will be done by it

c) the quantity of electricity on the positive plate will be greater than +Q

d) there will be no change in the energy stored in the capacitor

e) the quantity of electricity on the positive plate will be less than +Q

9. Standard resistance coils are usually wound double, that is, the coil is wound back on itself. This is done to

(5)

a) increase the resistance

b) decrease the resistance of the coil

c) minimize inductive effects in the coil

d) increase the IR drop in the coil

e) protect the meter with which the coil may be used

10. Very little sound comes from a violin string alone. More is produced when the string is fixed to a sounding box. The

(5) additional energy now appearing in the sound wave is accounted for by which of the following?

a) Thermal energy

b) The PE stored in the curved body of the sounding box whose walls are under tension.

c) Energy drawn from the surrounding atmosphere.

d) A mechanical advantage governed by the string length and the bridge height.

e) The increased energy supplied by the bow.

(Points)

11. A solid cylinder rolls down an inclined plane in time t. A hole is then bored at its geometric axis. The time now to roll down the plane is, compared to t:

(5)

a) greater
b) less
c) the same
d) governed by the size of the hole
e) indeterminate

12. A cylinder of radius R has a string wrapped around it. The free end of the string is held fixed and the cylinder allowed to "unroll" in a vertical line. The linear acceleration of the cylinder is:

(5)

a) g
b) gR
c) mg
d) 2/3g
e) 1/2g

13. A body is slightly denser than water and very compressible. When submerged and let go it will:

(5)

a) sink a ways and stop
b) rise to the surface
c) sink fast for a while, then slow up
d) sink faster and faster
e) stay where it is first submerged

14. In an astronomical telescope if the mirror diameter were doubled without altering the radius of curvature, all of the following occur except:

(10)

a) the image intensity would be 4 times as great
b) the overall magnification would be doubled
c) the f-number would be halved
d) spherical aberration would increase
e) the same photographic action could be obtained in 1/4 the previous exposure time

(Points)

15. The amount of heat passing through the bottom of a coffee pot on a gas stove is INDEPENDENT of:

(5)
a) the temperature gradient in the bottom of the pot.
b) the thickness of the material in the bottom of the pot.
c) the kind of material of which the pot is made.
d) the area of the bottom of the pot.
e) the amount of coffee in the pot.

16. Two men stand on opposite sides of a camp fire. They find it difficult to make each other heard. The reason for this comes most appropriately under the title of:

(5)
a) refraction
b) diffraction
c) interference
d) polarization
e) reverberation

QUANTUM MECHANICS

Qualitative

Do all the problems
One Hour
Closed Book

(Points)

(15) 1. Consider a model of liquid helium in which each atom is confined to a cube whose edge is the mean nearest neighbor distance. Taking the density of liquid helium to be 0.15 gm/cm^3, calculate the zero point energy per helium atom. Compare the result with the heat of vaporization per atom which is derivable from the heat of vaporization per mole of 0.020 kcal (1 kcal = 4.2×10^{10} erg). How do the zero point energy and heat of vaporization compare for an ordinary liquid such as water?

(10) 2. i) At what density must one begin to consider quantum effects in a (nonrelativistic) thermal plasma?

ii) Make a qualitative argument which gives the form of the ionization-recombination relation (Saha equation) between the various number densities in an equilibrium plasma.

(10) 3. Consider our sun squeezed down to the size of a neutron star, $R \sim 10$ km. Estimate roughly the Fermi energy of the nucleons in eV.

(15) 4. Consider two electrons in a one-dimensional box.

(a) Construct the lowest energy eigenstate, ignoring the Coulomb force.

(b) If we include the Coulomb force, the answer to (a) may not be the lowest energy state. What state is the best alternative candidate in this case?

Physics Qualifying Exam UCI October 1978

QUANTUM MECHANICS II

Open Book

Do all problems

Relative Weight		
10	1.	Consider a particle of mass m moving in the earth's gravitational field so that the potential energy is zero at the earth's surface $z = 0$. Draw $V(z)$. Calculate the (one dimensional) energy eigenvalues in the WKB approximation.
8	2.	A particle of mass m is scattered off of a potential $V = \beta\delta(\underline{r})$. Calculate the phase shifts $\delta_\ell(E)$ in the Born approximation. Plot the cross section σ versus energy E. From applying conservation of probability, at what energy must this approximation for σ be wrong.
6	3.	An $\ell = 2$ operator $Q^{\ell=2}$ has a matrix element squared

$$|\langle \psi_{j_2=2,m_2=1} | Q^{\ell=2}_{m=0} | \psi_{j_1=1,m_1=1} \rangle|^2 = 11 \quad .$$

Using the Wigner-Eckart Theorem and the attached Table of Clebsch-Gordan Coefficients, calculate

a) $|\langle \psi_{j_2=2,m_2=2} | Q^{\ell=2}_{m=1} | \psi_{j_1=1,m_1=1} \rangle|^2$

b) $|\langle \psi_{j_2=2,m_2=0} | Q^{\ell=2}_{m=0} | \psi_{j_1=1,m_1=0} \rangle|^2$.

Relative Weight

10

4. Consider an spin $\frac{1}{2}$ particle (mass m) with a magnetic moment $\underline{\mu} = \gamma\underline{S} = \frac{1}{2}\gamma\hbar\underline{\sigma}$ which moves through a uniform magnetic field (perpendicular to the motion) $\underline{B} = B_o\hat{z}$ for a distance L.

a) Calculate the phase change $\beta = (\Delta k)L$ for the particle going through this field. Δk is the change in $k(=p/\hbar)$ in the weak B field.

b) Calculate the precessional angle $\alpha = \omega t$ for the particle's spin after passing through the B field.

c) For a precessional angle of $\alpha = 2\pi$, what is β? Discuss <u>briefly</u> the significance of this result with regard to the spin $\frac{1}{2}$ nature of the particle.

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			
.	.			

1/2 × 1/2

m_1	m_2	1
		+1
+1/2	+1/2	1

m_1	m_2	1	0
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

m_1	m_2	1
		-1
-1/2	-1/2	1

1 × 1/2

m_1	m_2	3/2
		+3/2
+1	+1/2	1

m_1	m_2	3/2	1/2
		+1/2	+1/2
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3

m_1	m_2	3/2	1/2
		-1/2	-1/2
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3

m_1	m_2	3/2
		-3/2
-1	-1/2	1

2 × 1/2

m_1	m_2	5/2
		+5/2
+2	+1/2	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+2	-1/2	1/5	4/5
+1	+1/2	4/5	-1/5

m_1	m_2	5/2	3/2
		+1/2	+1/2
+1	-1/2	2/5	3/5
0	+1/2	3/5	-2/5

m_1	m_2	5/2	3/2
		-1/2	-1/2
0	-1/2	3/5	2/5
-1	+1/2	2/5	-3/5

m_1	m_2	5/2	3/2
		-3/2	-3/2
-1	-1/2	4/5	1/5
-2	+1/2	1/5	-4/5

m_1	m_2	5/2
		-5/2
-2	-1/2	1

3/2 × 1/2

m_1	m_2	2
		+2
+3/2	+1/2	1

m_1	m_2	2	1
		+1	+1
+3/2	-1/2	1/4	3/4
+1/2	+1/2	3/4	-1/4

m_1	m_2	2	1
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

m_1	m_2	2	1
		-1	-1
-1/2	-1/2	3/4	1/4
-3/2	+1/2	1/4	-3/4

m_1	m_2	2
		-2
-3/2	-1/2	1

2 × 1

m_1	m_2	3
		+3
+2	+1	1

m_1	m_2	3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	-1/3

m_1	m_2	3	2	1
		+1	+1	+1
+2	-1	1/15	1/3	3/5
+1	0	8/15	1/6	-3/10
0	+1	6/15	-1/2	1/10

m_1	m_2	3	2	1
		0	0	0
+1	-1	1/5	1/2	3/10
0	0	3/5	0	-2/5
-1	+1	1/5	-1/2	3/10

m_1	m_2	3	2	1
		-1	-1	-1
0	-1	6/15	1/2	1/10
-1	0	8/15	-1/6	-3/10
-2	+1	1/15	-1/3	3/5

m_1	m_2	3	2
		-2	-2
-1	-1	2/3	1/3
-2	0	1/3	-2/3

m_1	m_2	3
		-3
-2	-1	1

1 × 1

m_1	m_2	2
		+2
+1	+1	1

m_1	m_2	2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	-1/2

m_1	m_2	2	1	0
		0	0	0
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3

m_1	m_2	2	1
		-1	-1
0	-1	1/2	1/2
-1	0	1/2	-1/2

m_1	m_2	2
		-2
-1	-1	1

3/2 × 1

m_1	m_2	5/2
		+5/2
+3/2	+1	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	-2/5

m_1	m_2	5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	-1	1/10	2/5	1/2
+1/2	0	3/5	1/15	-1/3
-1/2	+1	3/10	-8/15	1/6

m_1	m_2	5/2	3/2	1/2
		-1/2	-1/2	-1/2
+1/2	-1	3/10	8/15	1/6
-1/2	0	3/5	-1/15	-1/3
-3/2	+1	1/10	-2/5	1/2

m_1	m_2	5/2	3/2
		-3/2	-3/2
-1/2	-1	3/5	2/5
-3/2	0	2/5	-3/5

m_1	m_2	5/2
		-5/2
-3/2	-1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J - j_1 - j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2 × 3/2

m_1	m_2	3
		+3
+3/2	+3/2	1

m_1	m_2	3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	-1/2

m_1	m_2	3	2	1
		+1	+1	+1
+3/2	-1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	-2/5
-1/2	+3/2	1/5	-1/2	3/10

m_1	m_2	3	2	1	0
		0	0	0	0
+3/2	-3/2	1/20	1/4	9/20	1/4
+1/2	-1/2	9/20	1/4	-1/20	-1/4
-1/2	+1/2	9/20	-1/4	-1/20	1/4
-3/2	+3/2	1/20	-1/4	9/20	-1/4

m_1	m_2	3	2	1
		-1	-1	-1
+1/2	-3/2	1/5	1/2	3/10
-1/2	-1/2	3/5	0	-2/5
-3/2	+1/2	1/5	-1/2	3/10

m_1	m_2	3	2
		-2	-2
-1/2	-3/2	1/2	1/2
-3/2	-1/2	1/2	-1/2

m_1	m_2	3
		-3
-3/2	-3/2	1

2 × 3/2

m_1	m_2	7/2
		+7/2
+2	+3/2	1

m_1	m_2	7/2	5/2
		+5/2	+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	-3/7

m_1	m_2	7/2	5/2	3/2
		+3/2	+3/2	+3/2
+2	-1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	-2/5
0	+3/2	2/7	-18/35	1/5

m_1	m_2	7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	-3/2	1/35	6/35	2/5	2/5
+1	-1/2	12/35	5/14	0	-3/10
0	+1/2	18/35	-3/35	-1/5	1/5
-1	+3/2	4/35	-27/70	2/5	-1/10

m_1	m_2	7/2	5/2	3/2	1/2
		-1/2	-1/2	-1/2	-1/2
+1	-3/2	4/35	27/70	2/5	1/10
0	-1/2	18/35	3/35	-1/5	-1/5
-1	+1/2	12/35	-5/14	0	3/10
-2	+3/2	1/35	-6/35	2/5	-2/5

m_1	m_2	7/2	5/2	3/2
		-3/2	-3/2	-3/2
0	-3/2	2/7	18/35	1/5
-1	-1/2	4/7	-1/35	-2/5
-2	+1/2	1/7	-16/35	2/5

m_1	m_2	7/2	5/2
		-5/2	-5/2
-1	-3/2	4/7	3/7
-2	-1/2	3/7	-4/7

m_1	m_2	7/2
		-7/2
-2	-3/2	1

2 × 2

m_1	m_2	4
		+4
+2	+2	1

m_1	m_2	4	3
		+3	+3
+2	+1	1/2	1/2
+1	+2	1/2	-1/2

m_1	m_2	4	3	2
		+2	+2	+2
+2	0	3/14	1/2	2/7
+1	1	4/7	0	-3/7
0	2	3/14	-1/2	2/7

m_1	m_2	4	3	2	1
		+1	+1	+1	+1
+2	-1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	-1/14	-3/10
0	1	3/7	-1/5	-1/14	3/10
-1	2	1/14	-3/10	3/7	-1/5

m_1	m_2	4	3	2	1	0
		0	0	0	0	0
+2	-2	1/70	1/10	2/7	2/5	1/5
+1	-1	8/35	2/5	1/14	-1/10	-1/5
0	0	18/35	0	-2/7	0	1/5
-1	1	8/35	-2/5	1/14	1/10	-1/5
-2	2	1/70	-1/10	2/7	-2/5	1/5

m_1	m_2	4	3	2	1
		-1	-1	-1	-1
+1	-2	1/14	3/10	3/7	1/5
0	-1	3/7	1/5	-1/14	-3/10
-1	0	3/7	-1/5	-1/14	3/10
-2	1	1/14	-3/10	3/7	-1/5

m_1	m_2	4	3	2
		-2	-2	-2
0	-2	3/14	1/2	2/7
-1	-1	4/7	0	-3/7
-2	0	3/14	-1/2	2/7

m_1	m_2	4	3
		-3	-3
-1	-2	1/2	1/2
-2	-1	1/2	-1/2

m_1	m_2	4
		-4
-2	-2	1

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2} \qquad d^1_{0,0} = \cos\theta$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. (Table extended April 1974.)

CLASSICAL MECHANICS

Instructions

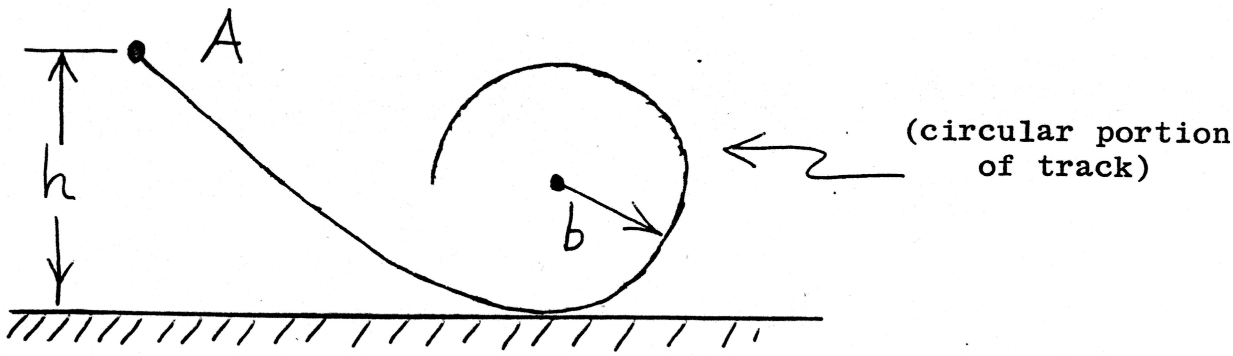
Do a total of three problems, as follows:

(1)	either #1 or #2	(30 points)
(2)	either #3 or #4	(35 points)
(3)	everyone do #5	(35 points)

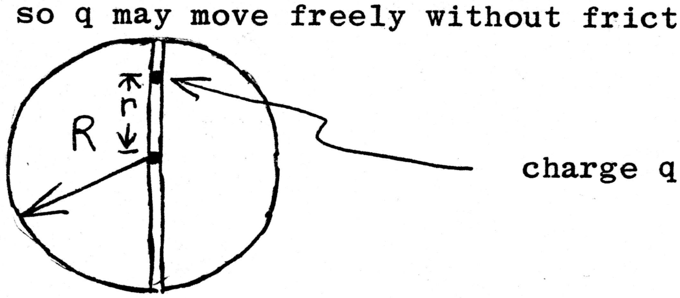
In grading, considerable attention will be given to the clarity and completeness of the solution. You can lose points if the solution appears to be random guesswork, with only a portion headed in the right direction.

Classical Mechanics

1. A small, uniform, solid sphere of radius r is initially released from rest at point A. Gravity acts downward, and the ball rolls on the track without slipping. How big must h be for the ball to roll completely around the track without falling off?



2. A positive electric charge of strength q is placed inside a uniformly charge sphere of radius R and total charge $Q>0$. The electrical charge is distributed uniformly throughout the volume of the sphere. A tiny hole is drilled along a diameter, so q may move freely without friction.

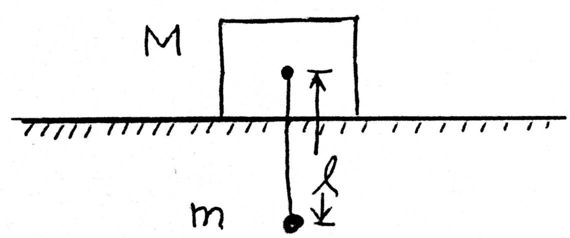


(15 points) (a) If q is given the velocity V_0 at $t = 0$, find an expression for its position as a function of time, for the portion of its trajectory when $0 < r < R$.

(15 points) (b) Say whatever you can that is simple about its motion when $r > R$, and in particular find its velocity at $r = \infty$ in terms of q, Q, R, and V_0.

3. An elementary result of mechanics is that a ball tossed vertically into the air strikes the ground again at the time $t_o = 2V_o/g$, where V_o is the ball's initial velocity and g is the acceleration of gravity.
Consider a ball tossed vertically into an atmosphere that opposes its motion with the frictional force $\vec{F} = -\eta\vec{v}$. How much longer does the ball stay in the air, if η is small? (More precisely, if t_o is the time the ball strikes the ground, what is the correction to the elementary formula first order in η?)

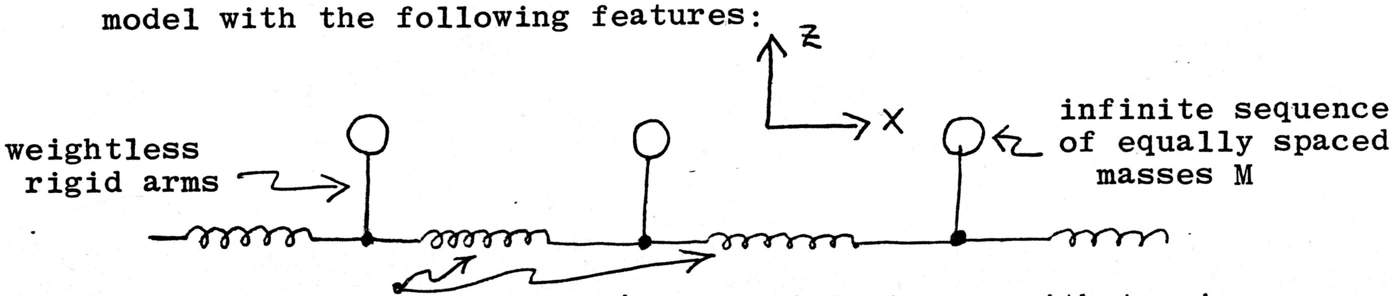
4. A mass M slides on a frictionless shelf, while a second mass m dangles below on a string of length ℓ. Gravity acts downward,



(15 points) (a) Set up a Lagrangian for the system, and find the equations of motion for the relevant dynamical variables.

(20 points) (b) The mass M is given a sharp, very brief impulsive blow at $t = 0$, so at $t = 0+$ it has velocity V_o. Assume the mass on the string never moves very far from the vertical, and find an expression for the position of M as a function of time. What is the <u>average</u> velocity with which M moves?

5. At a Physics Department colloquium last year, the speaker discussed in detail a number of properties of a physical model with the following features:



identical torsion fibers. Each resists torque with torsion constant k. Assume torque is linear with angle of twist.

Gravity acts downward.

(15 points) (a) Find the equation of motion obeyed by θ_i, the angle made by the i^{th} weightless arm with the vertical. THE MASSES MOVE ONLY IN THE PLANE PERPENDICULAR TO THE X AXIS.

(15 points) (b) Find the frequency of small amplitude waves in θ_i, as a function of wave vector, for the case where each mass points <u>upward</u> as shown. Comment on any unusual feature of the result.

(5 points) (c) Describe, very briefly, the subject of the colloquium where this example arose.

THERMODYNAMICS AND STATISTICAL MECHANICS

Do all 4 problems
Problems count equally
CLOSED BOOK

1. Consider an ideal gas with a specific heat ratio $\gamma = C_P/C_V$ which is constant. For a quasistatic adiabatic process, prove:

   a. That PV^γ = constant, where P is the pressure and V is the volume of the gas.

   b. That the work done W is given by

   $$W = \frac{P_iV_i - P_fV_f}{\gamma - 1} \quad ,$$

   where the subscripts i and f refer to the initial and final states, respectively.

   c. That the initial and final temperatures, T_i and T_f, are related by

   $$\frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1}$$

2. a. Derive the thermodynamic relation

   $$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

   where U is the internal energy.

   b. Apply this relation to black body radiation whose pressure P is one-third of the energy density u and derive the Stefan-Boltzmann law $u = bT^4$ where b is a constant.

3. An aqueous solution at temperature T contains a small concentration of magnetic ions with net spin $\frac{1}{2}$ and is placed in an inhomogeneous magnetic field pointing in the z-direction and having a magnitude specified by

$$H(z) = H(o) + \Delta z$$

where Δ is a constant.

a. Let $n_+(z)dz$ be the mean number of ions whose spin quantum number $s = +\frac{1}{2}$ and which are located between z and z + dz. What is the ratio $n_+(z_1)/n_+(o)$?

b. Let $n(z)dz$ be the total mean number of ions (of both directions of spin orientation) located between z and z +dz. What is the ratio $n(z_1)/n(o)$? If $\Delta > 0$ and $z_1 > 0$, is this ratio less than, equal to, or greater than unity?

4. The potential energy V of a linear chain of N atoms is given by

$$V = \sum_{n=1}^{N} \{\tfrac{1}{2}A(u_n - u_{n-1})^2 + B(u_n - u_{n-1})^3 + C(u_n - u_{n-1})^4\}$$

where u_n is the displacement of the nth atom from its equilibrium position. Assuming that B and C are small quantities, calculate the thermal expansion coefficient of the chain to first order in B and C. The following integrals may be useful:

$$\int_{-\infty}^{\infty} dx\, e^{-x^2} = \sqrt{\pi}\ , \qquad \int_{-\infty}^{\infty} dx\, x^2 e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

$$\int_{-\infty}^{\infty} dx\, x^4 e^{-x^2} = \frac{3\sqrt{\pi}}{4}$$

CLASSICAL MECHANICS

Open Book

Do 3 of the 4 Problems

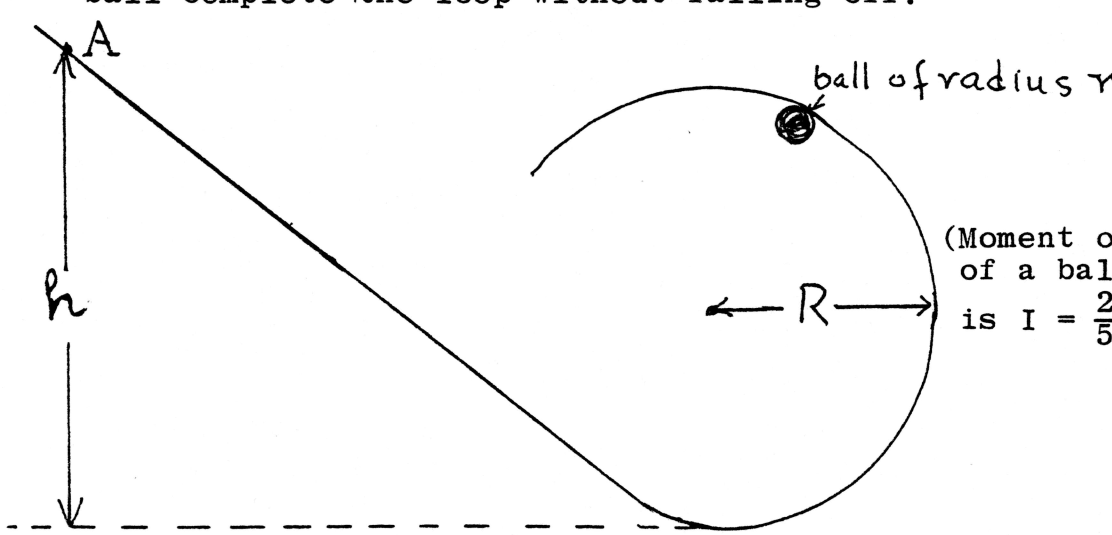
1. A relatively good model for the vibrations of a triatomic molecule of the type SO_2, NO_2, H_2O is given by three masses $m \neq M$ connected by elastic "springs" capable of longitudinal and transverse motions (with different spring constants, say k, K, respectively):

Assuming that the motion is two-dimensional and that there is no coupling between the longitudinal (x-direction) and transverse (y-direction) modes find:

a) the number of distinct vibration modes and their qualitative aspect;

b) the frequencies of the modes, assuming $M = 2m$, $K > k$.

2. A small uniform ball of radius r is released from rest at the point A of an inclined track. Assuming that the ball rolls without slipping determine the height h required to make the ball complete the loop without falling off.



(Moment of inertia of a ball of radius r is $I = \frac{2}{5} mr^2$.)

3. Use the Hamilton-Jacobi method to determine the motion of a two-dimensional harmonic oscillator:

a) Write down the Lagrangian and the Hamiltonian for the problem.

b) Write down the time-dependent Hamilton-Jacobi equation and find a complete integral.

c) Use Jacobi's theorem to determine the trajectories and the motion along them.

4. A rigid body has principal moments of inertia $I_1 < I_2 < I_3$ corresponding to the three principal axes with unit vectors $\hat{e}_1, \hat{e}_2, \hat{e}_3$.

If one adds a small mass ε at the point $\vec{q} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3$ show that the change in the value of I_1 is

$$I_1(\varepsilon) = I_1 + \varepsilon(x_1^2 + x_2^2)$$

and that the axis $\hat{e}_1$ changes to

$$\hat{e}_1(\varepsilon) = \hat{e}_1 + \varepsilon[x_1x_2\hat{e}_2/(I_2 - I_1) + x_1x_3\hat{e}_3/(I_3 - I_1)] \quad .$$

(Hint: Expand the kinetic energy to order ε.)

GENERAL PHYSICS

Logically discuss any 6 problems. Credit will be awarded only for the logical development steps toward a solution. In most cases, you need to identify the problem and then discuss.

1. $\Sigma^- \to n + \pi^-$

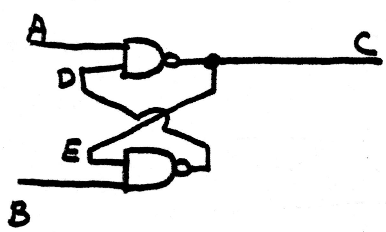
$m_n = 939.6$ MeV

$m_{\Sigma^-} = 1197.3$ MeV

$m_{\pi^-} = 139.6$ MeV

$\tau = 1.5 \times 10^{-10}$ sec.

2.



3. The index of refraction for hard x-rays in crystals is less than 1!

4. The words - 0, the words!

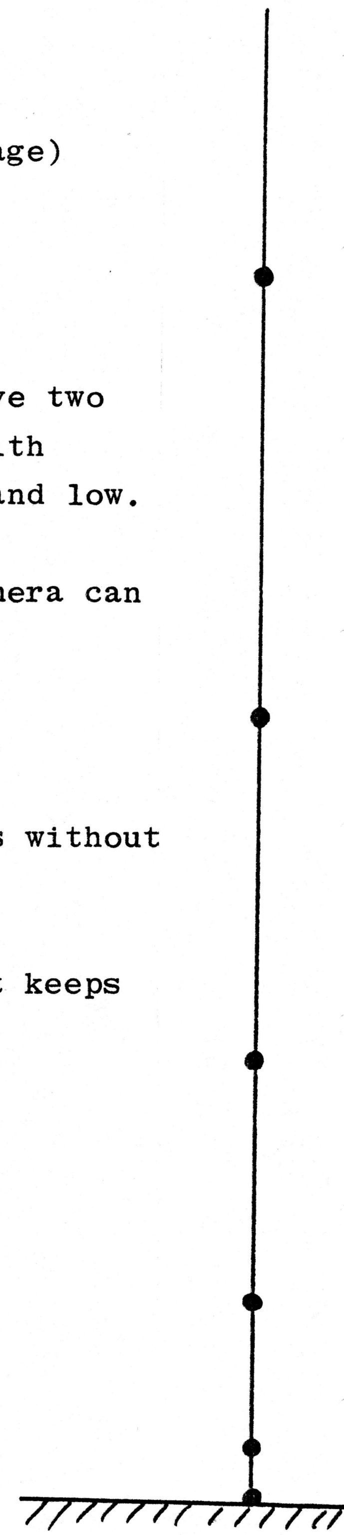
LCD	CRT
RAM	BCD
SQUID	LED
EPR	LASER
FWHM	
FET	

5. A 75 watt incandescent light has a coated glass bulb approximately 6 cm in diameter while the same type 150 watt bulb is approximately 9 cm in diameter.

6. Fusion of hydrogen in the sun produces helium plus ? .

7. A string with metal beads is held upright from the floor and then released.

(see illustration right side of page)



8. Many places along the Atlantic coast of the USA have two equal high tides and two equal low tides "daily" with approximately 6 hours and 13 minutes between high and low.

9. With the same ambient lighting, the f stop on a camera can be 1.4 or 8 to take a good picture.

10. "Summer of the gluons: Jets of evidence."

11. An audio amplifier accepts up to 1 mV input signals without distortion. A 2 mV signal has a distorted output.

12. A clock is thrown as a projectile upward so that it keeps maximum proper time!

ELECTRICITY AND MAGNETISM

Do all problems.

1. A charge Q is distributed uniformly along the z axis from $z = -a$ to $z = +a$. Find the potential at a point (r,θ) where $r > a$ as an expansion in solutions to Laplace's equation.

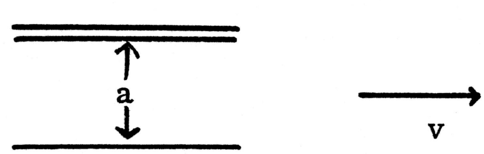
2. A "hard" ferromagnet is in the shape of a right circular cylinder of length L and radius a. The cylinder has a permanent magnetization M_o, uniform throughout its volume and parallel to its axis. The cylinder is oriented along the z axis from $z = -L/2$ to $z = L/2$, with magnetization along $\hat{z}$

   a) Determine the magnetic field $\bar{H}$ on the cylinder's axis above the cylinder.

   b) Expand for $z \gg L$, a and find the magnetic dipole moment.

   c) What is the effective magnetic current density and where is it located?

3. Two thin, parallel, infinitely long, nonconducting rods, a distance 'a' apart, with identical constant charge density λ per unit length in their rest frame, move with a relativistic velocity v, with respect to a stationary reference frame.



   a) Calculate the $\bar{E}$ and $\bar{B}$ fields and the force per unit length between the rods in a frame of reference moving with the rods.

b) Calculate charge and current densities in the stationary frame of reference.

c) Calculate the $\bar{E}$ and $\bar{B}$ fields in the stationary reference frame from the charge and current densities.

d) Show that these $\bar{E}$ and $\bar{B}$ in the stationary frame agree with those obtained by a Lorentz transformation of the fields from the comoving frame.

e) Calculate the force per unit length in the stationary frame and compare with that in the comoving frame.

MATHEMATICAL PHYSICS

Do all problems.

1. a. The function $f(z) = \dfrac{1}{(e^z + 1)^2}$ is expanded in a series of the form

$$f(z) = \sum_{n=0}^{\infty} a_n (z-1)^n \quad .$$

Discuss the convergence of the series.

b. Evaluate the integral

$$\oint_c \frac{dz(3z^3 + 2)}{(z-1)(z^2+9)}$$

where c is the circle $|z - 2| = 2$.

c. Through use of an appropriate contour, evaluate

$$\int_0^{\infty} \frac{dx\, \ln(x)}{1+x^2} \quad .$$

2. Given the differential operator

$$L = \frac{d^2}{dx^2} - K^2$$

which operates on the interval $0 \le x \le 3$. We want to solve

$$Lu = f$$

subject to the boundary conditions $u(o) = u(3) = 0$.

a. Construct the Green's function that may be used to generate the solution to the above equation.

b. For the choice $f(x) = 1$, obtain the explicit form of $u(x)$ through use of the Green's function.

3. Solve the integral equation

$$f(x) - \lambda \int_0^1 dy \sinh(x-y) f(y) = 1.$$

What values of λ are eigenvalues of the equation?

4. A fundamental principle of geometrical optics states that a light ray describes a trajectory which makes its transit time through matter an extremum. Mathematically, if $n(x,y)$ is the index of refraction at point (x,y), the path is such that

$$\int ds\, n(x,y)$$

is an extremum.

Consider the case where $n(x,y)$ depends on only x, $\lim_{x\to+\infty} n(x) = +1$, $\lim_{x\to-\infty} n(x) = n_o > 1$, and $n(x)$ is a monotonically decreasing function x.

A light ray moves through the medium, and as $x \to -\infty$ its trajectory is a straight line that makes an angle θ with the x axis. If $n_o \sin\theta < 1$, show the equation of its trajectory $y(x)$ is found from

$$y(x) = n_o \sin\theta \int^x \frac{dx'}{[n^2(x') - n_o^2 \sin^2\theta]^{\frac{1}{2}}} .$$

What happens to the ray when $n_o \sin\theta > 1$?

THERMODYNAMICS AND STATISTICAL MECHANICS

Two Hours. Open Book: One textbook & one math table allowed;
Do all problems

1. A system consists of three cylinders, the first containing one mole of a monatomic ideal gas at 300 K; the second, two moles of a monatomic ideal gas at 250 K; and the third, one mole of a diatomic ideal gas at 200 K. The characteristic rotational energy of the latter is 5 K, while its characteristic vibrational energy is 10,000 K.

   a) What is the maximum amount of work that can be extracted from the three bodies in a process in which all three are brought to a common final temperature?

   b) What is the final temperature, and

   c) What is the change in entropy in such a process of each subsystem? Of the total system?

2. Diatomic molecules get larger with increasing temperature for the same fundamental reasons that solids exhibit thermal expansion. Show that if the molecule is treated classically in the harmonic approximation the molecule has the same mean size at all temperatures. To obtain thermal expansion an anharmonic term is required:

$$- \epsilon r^3$$

where r is the displacement from equilibrium and ϵ is a small parameter. Find the lowest order contribution of the anharmonic term to the average size of the molecule, and to the specific heat.

3. In a <u>one</u> dimensional ferromagnetic solid the dispersion relation for spin waves is

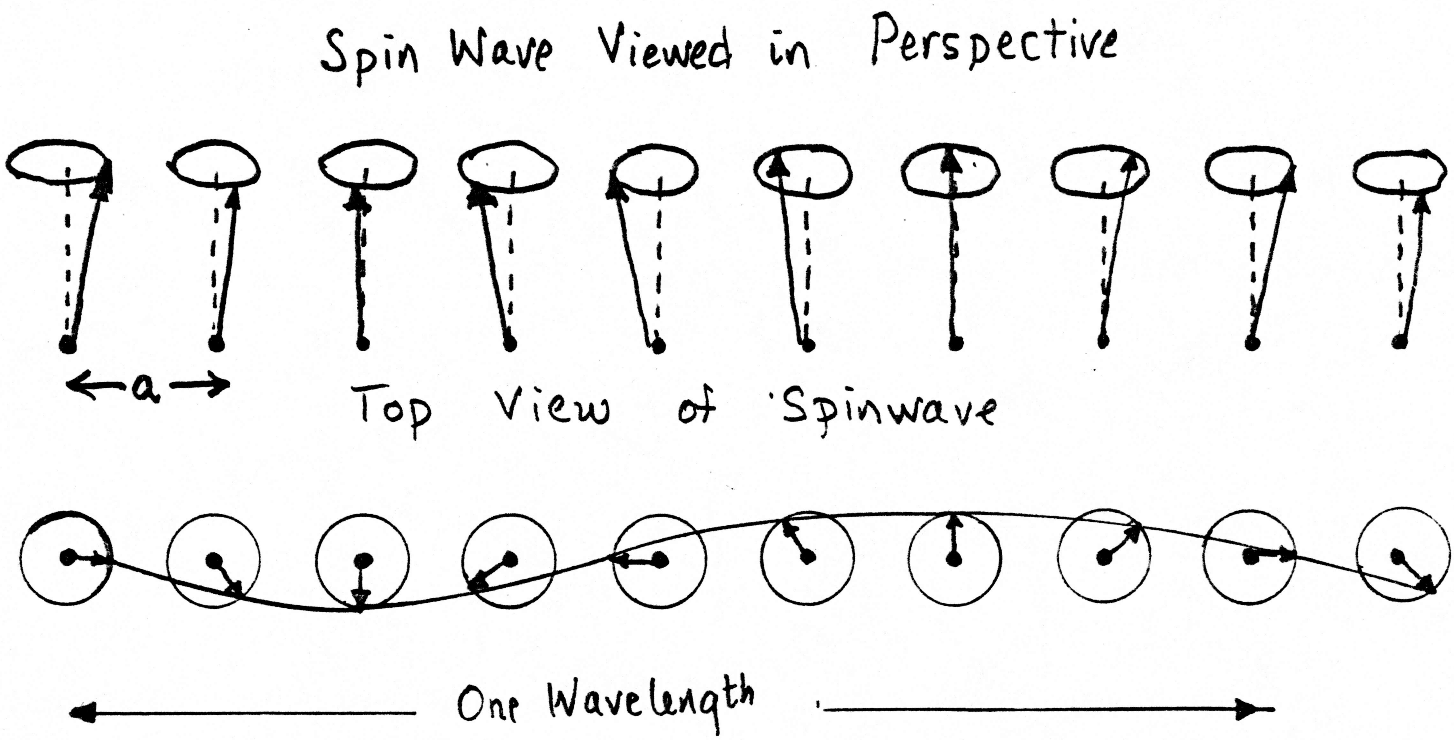
$$\hbar\omega = C(1 - \cos k a)$$

where C is a constant, k is the wave number, and a is the lattice constant. Such a wave is pictured below; note that the wavelength can't be smaller than the lattice constant. Assume that arbitrarily many spin waves in a given mode can be excited.

a) At low temperatures only the low energy modes are excited. Approximate the dispersion relation by its low energy form and find the appropriate cutoff frequency.

b) Assuming the dispersion relation and cutoff of a) give an exact expression for the specific heat. (What is the chemical potential, and why?)

c) Find the limiting form of the temperature dependence of the specific heat at low temperatures.



QUANTUM MECHANICS I

3 Hours
Open Book
Answer All Questions

1. A particle in three-dimensions is bound in a potential which gives rise to a large number of bound states. Among these the S-wave ($\ell = 0$) states are found to have energy levels E_n such that E_n is proportional to $n^{2/3}$ for large n. Find and demonstrate a form for the potential which yields this spectrum. (You may use a suitable approximation to justify your result.)

2. A particle is scattered by two weak scattering centers, A & B, with scattering amplitudes $f_A(\theta)$, $f_B(\theta)$, respectively. Suppose A and B, separated by a displacement $\vec{R}$, are placed simultaneously in an incident beam, with $\vec{R}$ making a well-defined but arbitrary angle with respect to the beam direction.

   a) Find an expression for the scattering amplitude in terms of f_A, f_B in lowest non-vanishing approximation.

   b) Find the form of the differential cross section in the limits of very large and very small incident wave length.

3. Consider two spin $\frac{1}{2}$ particles, A and B. B is essentially infinitely massive and attracts A with a harmonic oscillator potential of the form

$$V(r) = \tfrac{1}{2} m_A \omega^2 r^2 \quad .$$

In addition there is a spin-orbit coupling and an interaction between the spins, $\vec{S}_A$, $\vec{S}_B$. The interaction Hamiltonian is given by

$$H_1 = V(r) + \alpha \vec{L} \cdot \vec{S}_A + \beta \vec{S}_A \cdot \vec{S}_B$$

where

$$\omega \gg \hbar\alpha \gg \hbar\beta > 0 \quad .$$

Sketch the energy level diagram for all states $< \frac{3}{2}\hbar\omega$ above the ground state. Identify the levels completely by appropriate quantum numbers and give quantitatively the splittings between levels.

4. A system with spin $\frac{1}{2}$ and given g-factor is placed in a D.C. magnetic field of magnitude H_o along the z-axis, with spin initially "up" along the +z direction. At $t = 0$ an additional D.C. magnetic field of magnitude h is applied parallel to the x-axis and is left on for all $t > 0$. Find an explicit expression for the spin wave function for times $t > 0$. Describe the motion in words.

5. Two similar harmonic oscillators in one dimension have the same frequency and equilibrium position, and interact by means of a repulsive force which is proportional to the distance between them. Write and solve the Schrödinger equation for this system; determine the energy levels and specify the quantum numbers, including parity, which correspond to the levels.

QUANTUM MECHANICS II

1 hour
Closed Book
Answer 5 Questions

1. Describe several experiments which demonstrate the "wave-particle duality" of radiation and matter.

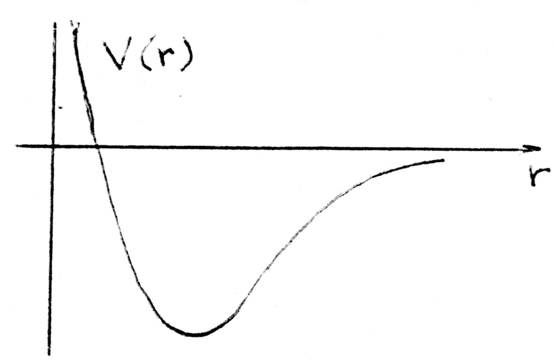
2. Describe briefly (giving specific examples) the key components of a laser and the basic principles underlying its operation.

3. a) Give the following properties and quantum numbers which correspond to the deuteron:
   (i) spin, (ii) parity, (iii) relative orbital angular momentum, (iv) isotopic spin, (v) magnetic moment (approximate).

   b) Give a relation between the binding energy of the deuteron and the quantity which characterizes the "radius" of the deuteron.

4. a) What is meant by the "natural line width" of an atomic spectroscopic line?
   b) What is meant by "Doppler broadening" of such a line?
   c) Give an estimate of Doppler broadening in a gaseous discharge at temperature T. Compare the magnitudes of the natural and Doppler widths expected for a typical optical line at a reasonable temperature.

5. A molecule consists of two atoms each of mass M. The potential energy as a function of interatomic distance is given roughly by the figure:
   Discuss the origin of the form of the potential, and give the low lying spectrum of the molecule.



6. Describe the Lamb Shift and the ground state hyperfine structure of hydrogen.

Useful constants:

$\hbar = 1.06 \times 10^{-27}$ erg-sec

$k_{Boltzmann} = 1.38 \times 10^{-16}$ erg $^{o}K^{-1}$

Physics Qualifying Exam UCI October 6, 1980

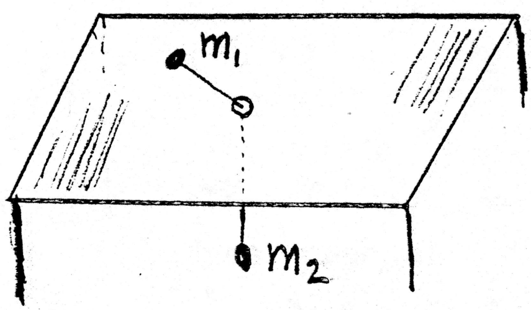
Classical Mechanics

Open Book

Do All Four Problems
Note: Problems have different point values.

30 Points

1. Two masses m_1 and m_2 are connected by a weightless string of length ℓ passing through a small hole in a horizontal table with m_2 hanging below the table; m_2 can move only in the vertical direction.



a) Choose an appropriate set of coordinates to describe this system and write the Lagrangian in terms of these.

b) Determine the equations of motion.

c) Under what conditions will m_2 be stationary?

d) If m_2 is slightly displaced from its stationary condition, small oscillations will ensue in the vertical direction. What will be the frequency of these oscillations?

20 Points

2. The Lagrangian for a particle is

$$L = \frac{1}{2} m \dot{\vec{r}}^2 + \dot{\vec{r}} \cdot (\vec{r} \times \vec{c})$$

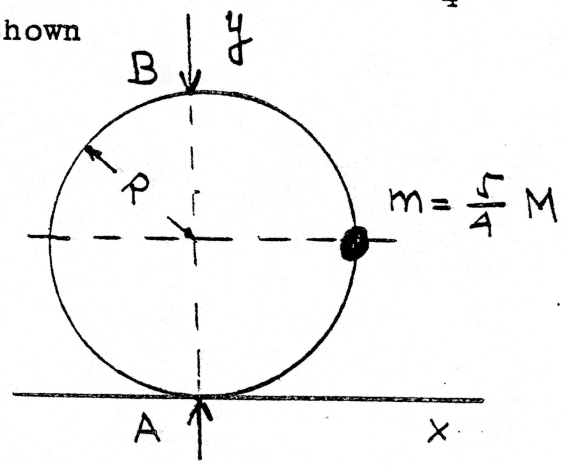
$\vec{c}$ is a given fixed vector

a) Find the equations of motion.

b) Find the Hamiltonian of this system.

30 Points

3. A thin disk of radius R and mass M lies in the x-y plane and and has a point mass $m = \frac{5}{4} M$ attached on its edge as shown



The moment of inertia tensor of the disk (without the extra mass point) about its center of mass is (z axis out of the paper)

$$I = \frac{MR^2}{4}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

a) Find the moment of inertia tensor of the disk about point A.

b) Find the moment of inertia tensor of the combination, mass point and disk, about A.

c) The disk is constrained to rotate about the y axis with angular velocity ω by pivots at A and B. Describe the angular momentum vector about A as a function of time and find the torque at A.

20 Points

4. Let $\Phi(q_i, P_j)$ be a generating function for a canonical transformation

$$Q_j = \frac{\partial \Phi}{\partial P_j}$$

Use the above to eliminate P_j in favor of Q_j and define

$$X(q_i, Q_j) = \sum_i Q_i P_i - \Phi$$

a) Find:

$$\frac{\partial X}{\partial q_i} = ?$$

$$\frac{\partial X}{\partial Q_j} = ?$$

b) Show that

$$\frac{\partial p_i}{\partial Q_j} = - \frac{\partial P_j}{\partial q_i}$$

Physics Qualifying Exam UCI October 6, 1980

Electricity and Magnetism

One Book: Jackson

15 points 1. A potential field is given by $\Phi_o(r,\theta,\varphi) = r^2 \sin 2\theta \cos\varphi$. Find the potential exterior to a grounded, conducting sphere of radius a placed in this field at the origin.

25 points 2. Write Maxwell's equations in media with $\bar{J}_{free} = 0$, $\rho_{free} = 0$.

a) Using the constitutive relations in terms of $\bar{P}$ and $\bar{M}$, rewrite Maxwell's equations with only $\bar{E}$, $\bar{B}$ terms on the left and only $\bar{P}$, $\bar{M}$ terms as bound sources on the right.

b) Write equations for the potentials $\bar{A}$, Φ in the Lorentz gauge using the $\bar{P}$, $\bar{M}$ terms as sources.

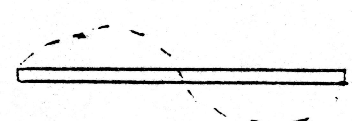
c) Now substitute for the potentials in terms of the vector fields $\bar{\pi}_e$, $\bar{\pi}_m$ defined by

$$\Phi = -\bar{\nabla}\cdot\bar{\pi}_e,\quad \bar{A} = \bar{\nabla}\times\bar{\pi}_m + \frac{1}{c}\frac{d\bar{\pi}_e}{dt}$$

d) Rearrange the equations in terms of a divergence and a curl equation and write second order equations for $\bar{\pi}_e$, $\bar{\pi}_m$ assuming the arguments of div and curl vanish.

e) Write the retarded solutions for $\bar{\pi}_e$, $\bar{\pi}_m$ in terms only of the sources $\bar{P}$, $\bar{M}$.

25 points 3. A linear antenna of length d is excited in such a way that the sinusoidal current makes a full wavelength of oscillation as shown in the figure.



a) Calculate the magnetic dipole moment.

b) Calculate the electric dipole moment.

c) Calculate the electric quadrupole moments.

d) What is the angular distribution of the power? (Hint: Use results in the text for this part.)

35 points

4. A parallel-plate capacitor consists of two circular plates, so that the system has an axis of symmetry. The radius is a, the plate separation is ℓ, and the material filling the space between the plates has dielectric constant ε. The capacitor is charged by being placed in a circuit that contains a source of emf V_o and a series resistor R. If the circuit is completed at time t = 0, find the following quantities within the capacitor as functions of the time:

b) the electric field

b) the magnetic field

c) Poynting's vector

d) the field energy

e) the scalar potential

f) the vector potential.

Neglect edge effects and assume RC >> a/c.

Physics Qualifying Exam UCI October 7, 1980

QUANTUM MECHANICS
3 Hours

Do 4 Problems
Textbook and classroom notes are OK.

1. Given a particle of mass m in a one-dimensional potential $V = \alpha|x|$ where α is a constant > 0, use some appropriate approximation to estimate the energy of the ground state and the first excited state.

2. Consider a harmonic oscillator interacting with a spin-$\frac{1}{2}$ system. The Hamiltonian is

$$H = \hbar\omega\, a^+ a + \hbar\omega\left(\frac{1+\sigma_z}{2}\right) + \hbar\lambda[a^+\sigma_- + a\sigma_+]$$

where as usual a, a^+ are annihilation and creation operators for the oscillator;

$$\sigma_\pm \equiv \frac{\sigma_x \pm i\sigma_y}{2}$$

with $\sigma_x, \sigma_y, \sigma_z \equiv$ Pauli spin matrices.

(i) Find an operator (other than H) which corresponds to a conserved quantum number of the system.

(ii) Suppose we start at $t = 0$ in a state with $n > 0$ quanta in the oscillator and spin pointing down (i.e. along $-z$ direction). Find the probability that at some later time t the spin is pointing up.

3. A particle is bound in a finite-range central potential $V(r)$ about some fixed point. The ground state and first excited S-state wave functions are φ_o and φ_1, respectively. The particle carries an electric charge. A second charged particle, which does not interact with V, is incident with wave vector $\vec{k}$.

3. Continued

a) Find an expression in first Born approximation for the differential cross section for scattering leading to excitation of the bound system from φ_o to φ_1.

b) Suppose the range of the potential V (and hence the extension of the wave functions φ_o and φ_1) is very much smaller than the wavelength of the incident beam. What will be the approximate angular dependence of the differential cross section in (a)?

c) What is the answer to the question in (b) if, instead of exciting the bound system, the scattering leaves it in the ground state φ_o?

4. Two spin-$\frac{1}{2}$ particles are bound together by an interaction consisting of a central potential, i.e., depending only on the inter-particle distance, and an effective spin-orbit interaction which we take to be

$$a_1 \vec{L} \cdot \vec{S}_1 + a_2 \vec{L} \cdot \vec{S}_2$$

Here $\vec{L}$ is the relative orbital angular momentum; $\vec{S}_1$ and $\vec{S}_2$ are the two spin operators; and a_1 and a_2 are constants which are almost, but not quite, equal. Find the structure of the energy levels for S and P states to zeroth order and first order in the small parameter $\epsilon \equiv a_1 - a_2$.

5. A spinless particle is scattered from a repulsive potential with $V(r) = V_o$ for $r < a$

$= 0$ for $r > a$

a) Find an expression for the S-wave scattering phase shift if the energy of the incident particle is less than V_o.

b) Use this result to find the scattering cross section at low energy.

Physics Qualifying Exam UCI October 7, 1980

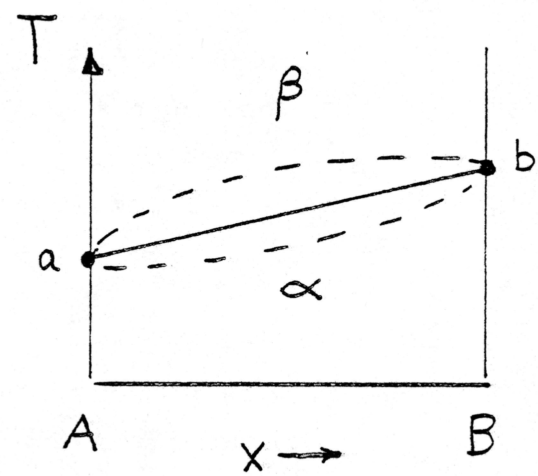
STATISTICAL MECHANICS

Open Book: two texts and one math tables book

Three Hours

Work either problem 1 or 2 and both problems 3 & 4.

1. Consider a low temperature phase transition in a metallic alloy $A_{1-x}B_x$, as pictured here in the $P = 0$ plane, between a high temperature (β) phase and a low temperature (α) phase. The dotted loop shows what the co-existence curve would look like if the two phases were allowed to adjust to equality of chemical potential. However, diffusion in solids requires thermal activation; hence at low temperatures the atoms are inhibited from moving around, and x is constrained to remain equal throughout the solid. Thus, the two phases coexist along a single, solid line ab. Find a formula analogous to the Clausius-Clapeyron equation for the slope of the solid line, $dx/dT|_{ab}$.



2. A model for salad dressing

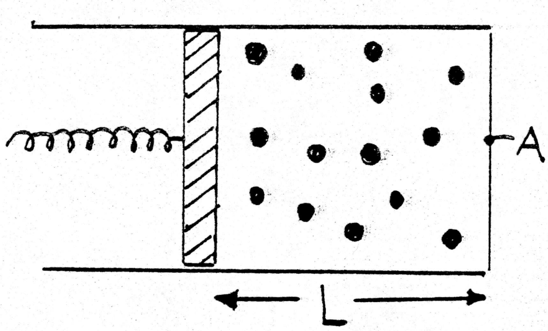
A certain oil has Gibbs free energy $G_o(T)$ equal to that of vinegar, i.e., $G_o(T) = G_v(T)$ at all temperatures. When the two are mixed there is a repulsion between oil and vinegar atoms such that the total free energy per atom for a mixture whose oil concentration is constrained to the value x is

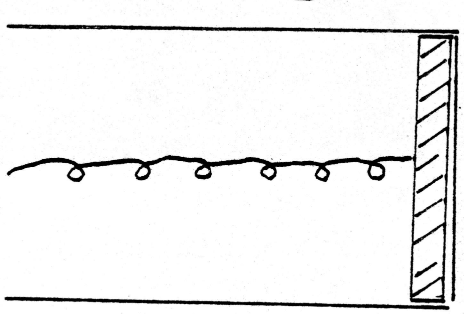
$$G = x\,G_o + (1-x)G_v + E_{ov}\,x\,(1-x) - RT\,S_{mix}$$

where the interaction E_{ov} is positive. What is the entropy of mixing S_{mix}? Using the appropriate minimization

2. (Continued)
principle and graphical analysis of the resulting equations show that there exists a critical temperature T_c below which there are two equilibirium choices of x. Estimate T_c and draw the xT phase diagram exhibiting the two phase region ("Miscibility gap".) Physically what happens to the mixture when our goodly gourmet graduate student chef prepares his salad dressing in the hot, hot desert, but then carries the jar up to the cold, cold snowcapped peaks?

3. A system of N monatomic, noninteracting atoms is contained in a cylinder of area A, enclosed by a weightless piston which is attached to the bottom of the cylinder by a spring. The spring is under zero tension when the volume is zero.
a) Write down the two equations of state for U(T,V,N) and P(T V N) in terms of the equilibrium volume.
b) Find the equilibrium volume by two methods:
   i) using a thermodynamic minimization principle, and
   ii) by statistical evaluation, using the L dependent partition function.
c) Evaluate the fluctuation in the volume at equilibrium





Zero tension configuration

4. A large evacuated chamber is filled with noninteracting, massless spin $\frac{1}{2}$ fermions; these are enclosed by walls through which the fermions cannot penetrate, but which act as a source and sink where the fermions are created and annihilated in such a way as to bring the fermions in the cavity to thermal equilibrium with the walls at temperature T. To what well-known problem is the situation analogous and what is the difference? What is the fermion chemical potential, and why? What is the energy per unit volume in the cavity? What is the pressure?

Physics Qualifying Exam UCI October 8, 1980

MATHEMATICAL PHYSICS

One book plus class notes.

1. Consider the ordinary differential equation

$$x(2-x)y'' + 2(1-x)y' + y = 0$$

a) What are the singular points of this equation? (5 points)

b) Find the first three terms in the power series expansions of two independent solutions about the origin. (20 points)

2. Evaluate the following integrals

a) $$\int_0^\infty \frac{dx}{1+x^5}$$ ($12\frac{1}{2}$ points)

b) $$\int_0^\infty \frac{\cos ax}{b^2+x^2}\, dx$$ ($12\frac{1}{2}$ points)

3. Solve the integral equations

a) $$f(x) = x + \lambda \int_0^1 y(x+y)\, f(y)\, dy$$ ($12\frac{1}{2}$ points)

b) $$f(x) = \lambda \int_0^1 \sin \pi(x-y)\, f(y)\, dy$$ ($12\frac{1}{2}$ points)

In case b), what are the eigenvalues of λ?

4. A uniform string of length 5 meters hangs from two supports at the same height, 3 meters apart. By minimizing the potential energy of the string, find the equation describing the curve it forms and determine the vertical distance between the supports and the lowest point of the string. (25 points)

General Physics

(Closed Book; you may use a table of physical constants.)

1. Explain why the paramagnetic susceptibility of metals is independent of temperature, rather than inversely proportional to temperature, as would follow from Curie's law, and why the magnitude of the susceptibility is much smaller than expected from that law.

2. Explain the meaning and/or origin of 6 of the following terms:
   a. Landau damping.
   b. Color confinement.
   c. Reynolds' number.
   d. Microwave background radiation.
   e. Soliton.
   f. Critical opalescence.
   g. Bohm-Aharonov effect.
   h. Van der Waals forces.
   i. Heat pump.
   j. Maser.
   k. Einstein-Podolsky-Rosen paradox.

3. Give a list of the quantum numbers (spin, parity, orbital momentum, isospin) for the ground state of the deuteron.

4. What is the relation between the lifetime of a resonant state and the width of the energy level describing it.

5. List the discoveries for which the following physicist have received Nobel prizes (and give the year, if you know it).
   a. N. Basov
   b. F. Bloch
   c. L. Esaki
   d. A. Hewish
   e. J. H. D. Jensen
   f. A. Kastler
   g. P. Kusch

5. Continued

   h. W. Pauli

   i. A. Penzias

   j. B. Richter

   k. R. Schrieffer

   l. I. E. Tamm

6. Why is the light coming from a clear sky polarized? Where is the maximum polarization located relative to the sun?

7. Give a brief explanation of the following fact:
A negative muon may replace an electron in an atom of arbitrary Z and move on a "Bohr orbit" according to the same laws as the electron ($m_\mu = 207\ m_e$). The transition energies from states with $n \leq 4$ to more strongly bound states in such mesic atoms are the same for neutral atoms as for atoms from which all electrons have been removed.

8. Discuss, without necessarily deriving, the scattering of a particle from a hard sphere of radius R. Compare the total cross sections in the low and high energy limits with the geometric cross section πR^2. Explain the high energy result, i.e., discuss the concept of "shadow scattering."

9. "Heathcliff gazed intently at the spectacular sunset, his attention absorbed by the beautiful rainbow surrounding the fiery globe."

   Aside from the wretched quality of the prose, what is wrong with the preceding paragraph?